MTH 114
ALGEBRA AND ELEMETARY TRIGONOMETRY LECTURE NOTES

for ND

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COURSE OUTLINE:

1. Understand the laws of indices and their application in simplifying algebraic expressions.
2. Understand the theory of logarithms and surds and their applications in manipulating expressions.
3. Understand principles underlying the construction of charts and graphs.
4. Know the different methods of solving quadratic equations.
5. Understand permutation and combination
6. Understand the concept of set theory
7. Understand the properties of arithmetic and geometric progressions
8. Understand the binomial theorem and its application in the expansion of expressions and in approximations.
9. Understand the basic concepts and manipulation of vectors and their applications to the solution of engineering problems.
10. Understand the concept of equations and methods of solving different types of equations and apply same to engineering problems.
11. Understand the definition, manipulation and application of trigonometric functions.

Contents

ASSESSMENT: The continuous assessment, tests and quizzes will be awarded 40% of the total score. The end of the Semester Examination will make up for the remaining 60% of the total score.

Chapter 1

Understand the laws of indices and their application in simplifying algebraic expressions.

1.1 Define index

Consider the following expression

$$2^5$$

the number 5 is called an index (or power) while the number 2 in the expression is known as the base.

1.2 Establish the laws of indices

The following are the laws of indices:

1. $a^m \times a^n = a^{m+n}$ e.g $2^5 \times 2^8 = 2^{5+6} = 2^{11}$, $3^8 \times 3^9 = 3^{8+9} = 3^{17}$ etc.
2. $a^m \div a^n = a^{m-n}$ e.g $3^5 \div 3^8 = 3^{5-6} = 3^{-1}$, $2^{17} \div 2^9 = 2^{17-9} = 2^8$ etc.
3. $(a^m)^n = a^{m \times n}$ e.g $(2^4)^5 = 2^{4\times5} = 2^{20}$
4. $a^{-n} = \frac{1}{a^n}$ e.g $3^{-2} = \frac{3^{-2}}{1} = \frac{1}{3^2}$, $4^5 = \frac{4^5}{1} = \frac{1}{4^{-5}}$ etc.
5. $a^{\frac{1}{n}} = \sqrt[n]{a}$ e.g $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$, $9^{\frac{1}{2}} = \sqrt{9} = 3$ etc.
6. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ e.g $2^{\frac{5}{2}} = \sqrt{2^5} = (\sqrt{2})^5$, $3^{\frac{3}{5}} = \sqrt[5]{3^3} = (\sqrt[5]{3})^3$ etc.
7. $a^0 = 1$ e.g $10^0 = 1$, e.g $387^0 = 1$ etc

1.3 Solve simple problems using the laws of indices.

Examples:

1. Simplify the following
   a. $27^{\frac{1}{3}}$
   b. $8^{-\frac{2}{3}}$
   c. $\frac{4^3}{2^2}$
   d. $8^2 \times 16^2$
   e. $8 \div 64$

2. Find the values of x in the following equations
   a. $25^{x+1} = 0.2$
   b. $81^{2x} = \frac{1}{27}$
   c. $5^{x+1} = 0.2$
   d. $4^x = 2^{3x+1}$
   e. $4^{2x} = 32^{12x-6}$
   f. $3^{2x} + 4(3^x) + 1 = 0$

3. Simplify
   a.
$$\left(\frac{16^{\frac{1}{4}} \cdot 8^{\frac{2}{3}}}{27^{\frac{2}{3}} \cdot 4}\right)$$
   b.
$$\left(\frac{3^{-2}6^2\sqrt{48}}{\sqrt{27}36^{-7}}\right)$$
   c.
$$\left(\frac{a^{-4}b^6}{a^{20}b^{41}}\right)^2$$
   d.
$$\left(\frac{x^{-1}y^2}{x^4y^{40}}\right)^2$$

Chapter 2

Understand the theory of logarithms and surds and their applications in manipulating expressions.

2.1 Define logarithm

Consider the equation

$$10^3 = 1000$$

this can be written in log form as

$$\log_{10} 1000 = 3$$

Another example is; if

$$5^2 = 25$$

then

$$\log_5 25 = 2$$

The equation $5^2 = 25$ is called an index equation and the corresponding equation $\log_5 25 = 2$ is called a logrithmic equation. Therefore, the log of a number x to base a, is a number n such that $x = a^n$

2.2 Establish the four basic laws of logarithm

The following are the 4 basic laws of logarithm

1. $\log_a P + \log_a Q = \log_a(P \times Q)$ e.g $\log_4 9 + \log_4 21 = \log_4(9 \times 21) = \log_4 189$ etc.

2. $\log_a P - \log_a Q = \log_a(P \div Q)$ e.g $\log_7 81 - \log_7 9 = \log_7(81 \div 9) = \log_7 \frac{81}{9} = \log_7 9$ etc.
3. $\log_a P^n = n \log_a P$ e.g $\log_2 12^3 = 3 \log_2 12$ etc.
4. $\log_a P = \frac{\log_c P}{\log_c a}$ e.g $\log_5 20 = \frac{\log_3 20}{\log_3 5}$

2.3 Solve simple logarithm problem

Examples:

1. Simplify
   a. $\log_3 27$
   b. $\log_7 49$
2. Simplify
   a. $log_5 24 - log_5 15 + log_5 100$
   b. $log_7 98 - log_7 30 + log_7 15$
   c. $log_5 \sqrt{6} + log_5 9$
3. Find x if $\log_x(5x - 6) = 2$
4. Given that $\log 5 = 0.6990$ and $\log 3 = 0.4771$, simplify

$$\log 15 + \log 45 - \log 27$$

5. Simplify $\log_5 13.5$

2.4 Define natural logarithm and common logarithm.

2.4.1 Common Logarithm

Logarithm to base 10 ($\log_{10}$) are used to reduce the labour in numerical calculations. These logs (to base 10) are called "common log". The common logs are usually found in tables (four figure tables).

2.4.2 Natural Logarithm

There are also special logarithm. They are logarithm taken to base $e = 2.7138....$ They are represented by the symbol ln i.e. $ln5 = \log_e 5$.

2.5 Apply logarithm in solving non-linear equations.

Logarithm can be used to simplify nonlinear equations which are sometimes difficult to solve. The simplification converts the equation from a nonlinear to a linear equation which is much more easier to solve. The following are some examples of nonlinear equations and how they are simplified using logarithm.

2.5.1 The equation $y = ax^n$

Consider the equation

$$y = ax^n$$

to simplify this we take the log of both sides,

$$\begin{aligned} \log y &= \log ax^n \\ &= \log a + \log x^n \\ &= \log a + n \log x \end{aligned}$$

If we let $Y = \log y, A = \log a$ and $X = \log x$, then we obtain the following linear equation

$$Y = A + nX$$

which is much easier to solve

Examples: Convert the following nonlinear equations to linear equations using logarithm

1. $y = bc^x$
2. $y = ax^5$
3. $y = 100x^2$ hence find y when $x = 10$
4. $y = 10x^{100}$ hence find y when $x = 5$
5. $y = 25x^5$ hence find y when $x = 2$

2.6 Define surds

Surds are irrational numbers. They are roots of numbers like $3, 5, 7, 11$ etc. Examples of surds include $\sqrt{3}$, $\sqrt{5}$, $\sqrt{34}$, $\sqrt{21}$, $\sqrt{71}$ etc. Any number that contains a surd is also a surd for example, $3+\sqrt{3}, 4\sqrt{10}, 8-\sqrt{7}$ etc.

2.6.1 Rules of Surds

The following are the 2 basic rules of surds

1. $\sqrt{m} \times \sqrt{n} = \sqrt{m \times n}$ e.g $\sqrt{5} \times \sqrt{2} = \sqrt{5 \times 2} = \sqrt{10}$
2. $\sqrt{m} \div \sqrt{n} = \sqrt{m \div n}$ e.g $\sqrt{100} \div \sqrt{4} = \sqrt{100 \div 4} = \sqrt{25} = 5$
   Note that: $\sqrt{m} \pm \sqrt{n} \neq \sqrt{m \pm n}$ and

2.7 Reduce a surd into its simplest form

Reducing a surd into its simplest form is done by removing any factor of the number under the root sign which is a perfect square (has a root). This is illustrated using the following examples

Examples: Reduce the following surds to their simplest form

1. $\sqrt{8}$
2. $\sqrt{72}$
3. $\sqrt{50}$
4. $\sqrt{162}$
5. $\sqrt{27}$

2.8 Solve simple problems on surds

2.8.1 Algebra of Surds

1. Addition/Subtraction: When adding or subtraction surds, like terms are grouped together and added/subtracted.

Examples: Simplify

(a) $2+\sqrt{3}+3+5\sqrt{3}$

(b) $4\sqrt{3}-5\sqrt{2}+2\sqrt{3}+8\sqrt{2}$

(c) $23\sqrt{5}-8+23\sqrt{5}-40$

(d) $2-12\sqrt{2}+80+\sqrt{5}$

(e) $1-\sqrt{27}-3+\sqrt{3}$

2. Multiplication: Surds are multiplied in the normal way. You should note that $\sqrt{a}\times\sqrt{a}=(\sqrt{a})^2=a$

Examples: Simplify

(a) $(2+\sqrt{3})(3+5\sqrt{3})$

(b) $(4\sqrt{3}-5\sqrt{2})(2\sqrt{3}+8\sqrt{2})$

(c) $\sqrt{5}(8+23\sqrt{5})$

(d) $(2-12\sqrt{2})(80+\sqrt{5})$

(e) $(1-\sqrt{3})(1+\sqrt{3})$

3. Rationalization: When surd appears in the denominator of a fraction, we can rationalize it by multiplying the fraction by another fraction whose numerator and denominator are the same and the conjugate of the denominator of the surd we want to rationalize. The conjugate of a surd is obtained by changing the sign of the surd for example, the conjugate of the surd $2+\sqrt{3}$ is $2-\sqrt{3}$ and that of $\sqrt{3}-4$ is $-\sqrt{3}-4$.

Examples: Rationalize the denominator of the following surds

(a) $\frac{2}{2+\sqrt{3}}$

(b) $\frac{4}{3+8\sqrt{2}}$

(c) $\frac{\sqrt{5}}{5-\sqrt{5}}$

(d) $\frac{2+\sqrt{2}}{2-\sqrt{2}}$

(e) $\frac{1-\sqrt{3}}{1+\sqrt{3}}$

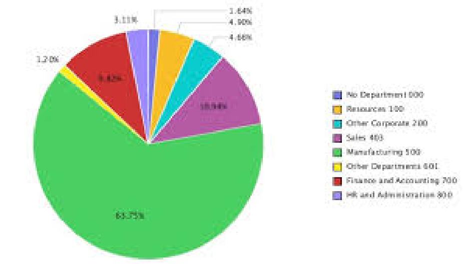
Chapter 3

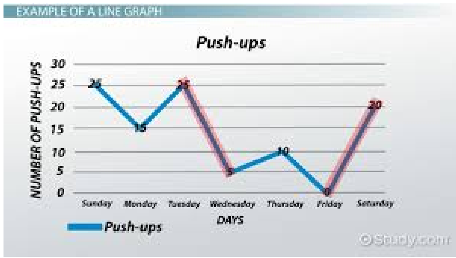
Understand principles underlying the construction of charts and graphs.

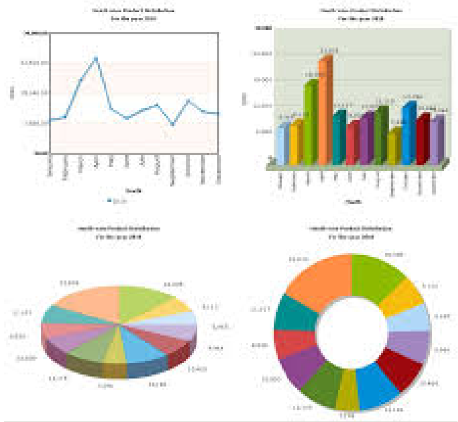
3.1 Definition

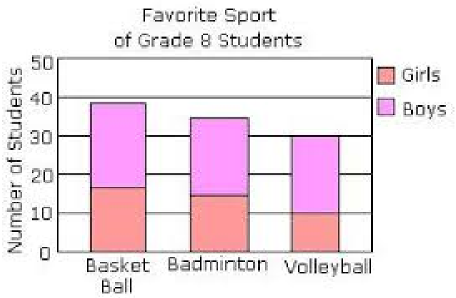
3.1.1 Chart

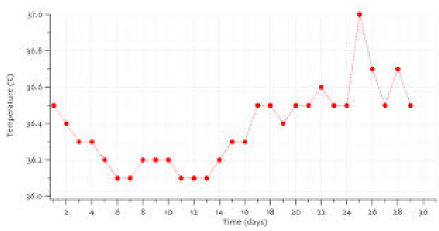
A chart is a graphical representation of data. It is a diagram, picture, or graph which is intended to make information easier to understand. It can also be defined as a drawing that shows information in a simple way, often using lines and curves to show amounts. Examples include:











3.1.2 Graph

A graph is a diagram showing the relation between variable quantities, typically of two variables, each measured along one of a pair of axes at right angles. It is a series of points, forming a curve or surface, each of which represents a value of a given function.

3.2 Construct graphs of functions fractions such as Y = ax +b,n = 1,2 Y = CST (a+x) Y = axk, including cases of asymbles

3.2.1 Graph of $y = ax + b$

To construct the graph of $y = ax + b$ we need to make a table of values of x and corresponding values of y. For example, suppose we want to draw the graph of the function

$$y = 2x + 6$$

for the values $x = -2, -1, 0, 1, 2, 3$, we will create a table of values of x and corresponding values of y. Firstly, lets find all values of y using the value $x = -2, -1, 0, 1, 2, 3$

x	Corresponding $y = 2x + 6$ value
-2	$y = 2(-2) + 6 = -4 + 6 = 2$
-1	$y = 2(-1) + 6 = -2 + 6 = 4$
0	$y = 2(0) + 6 = 0 + 6 = 6$
1	$y = 2(1) + 6 = 2 + 6 = 8$
2	$y = 2(2) + 6 = 4 + 6 = 10$
3	$y = 2(3) + 6 = 6 + 6 = 12$

so we obtain the following table

x	-2	-1	0	1	2	3
y	2	4	6	8	10	12

We now proceed to plotting these point on a graph sheet

Examples: Draw the graph of the following functions using the indicated values for x

1. $y = x^2 - x - 6$ for $x = -3, -2, -1, 0, 1, 2, 3, 4, 5$
2. $y = x^2 - 5x + 6$ for $x = -1, 0, 1, 2, 3, 4, 5$
3. $y = 2x^3$ for $x = -3, -2, -1, 0, 1, 2, 3, 4, 5$
4. $y = x^4$ for $x = -3, -2, -1, 0, 1, 2, 3, 4, 5$
5. $y = 3x + 7$ for $x = -3, -2, -1, 0, 1, 2, 3, 4, 5$

3.3 Apply knowledge from 3.1 in determination as laws from experimental data.

Examples:

1. Draw the graph of $y = 2x + 4$ for $x = -2, -1, 0, 1, 2, 3$ and find the slope of the line
2. Draw the graph of $y = x^2 - 5x + 6$ for $x = -2, -1, 0, 1, 2, 3, 4, 5$ and find the slope at $x = 2$
3. Draw the graph of $s = t^2 - t - 6$ for $t = -2, -1, 0, 1, 2, 3, 4, 5$ and find the slope at $t = 2$
4.
5.

Chapter 4

Know the different methods of solving Quadratic Equations (QE).

4.1 Quadratic Equation

A quadratic equation is any equation of the form

$$ax^2 + bx + c = 0$$

where a, b and c are constants with $a \neq 0$. For example
$2x^2 + 3x + 4 = 0,\ a = 2, b = 3, c = 4$
$x^2 - 5x + 4 = 0,\ a = 1, b = -5, c = 4$
$12 - 3x + 5x^2 = 0,\ a = 5, b = -3, c = 12$
$7x^2 + 3x = 0,\ a = 7, b = 3, c = 0$
$2x^2 - 4 = 0,\ a = 2, b = 0, c = -4$
$8x^2 = 0,\ a = 8, b = 0, c = 0$

4.1.1 Roots (solution) of a Quadratic Equation

The root of a quadratic equation is a value of x which when substituted into the equation, the equation is satisfied. For example, 2 is a root of the quadratic equation $x^2 - 5x + 6 = 0$ because, if 2 is substituted into the equation, the equation is satisfied i.e.

$$\begin{aligned} x^2 - 5x + 6 &= 2^2 - 5(2) + 6 \\ &= 4 - 10 + 6 \\ &= 0 \end{aligned}$$

hence the equation $x^2 - 5x + 6 = 0$ is satisfied.
Every quadratic equation has 2 roots which are obtained by solving the equation. There are 4 different methods of solving quadratic equations which will be discussed in the following sections.

4.2 Solve quadratic equations by factorization

This method uses the idea of factorization to solve quadratic equations. Not that not all quadratic equations can be solved using this method because some quadratic equations are not factorizable. In this case, solving a non factorizable equation requires using other methods. The following steps are followed when solving quadratic equation using factorization method:

Step 1: Convert the equation to the form: $ax^2 + bx + c = 0$ if it is not in this form

Step 2: Check the value of a

a. If $a = 1$, then look for 2 numbers (factors) m and n. such that: $m + n = b$ and $m \times n = c$

b. If $a \neq 1$, then look for 2 numbers (factors) m and n. such that: $m + n = b$ and $m \times n = a \times c$

Step 3: Attach x (or the variable in the Q.E.) to the factors to obtain mx and nx

Step 4: Replace bx in the initial equation by $mx + nx$ to obtain: $ax^2 + mx + nx + c = 0$

Step 5: Group the resulting equation in the form $(ax^2 + mx) + (nx + c) = 0$ and factorize what is common in each group. This results to an expression of the form: $(x + \alpha)(x + \beta) = 0$

Step 6: Equate each bracket to zero and find the value of x i.e. either $(x + \alpha) = 0$ or $(x + \beta) = 0$. And this results to 2 values for x which are the roots of the QE.

Examples:

1. $2x^2 + 3x + 4 = 0$
2. $x^2 + 3x + 2 = 0$
3. $6x^2 + 11x + 4 = 0$
4. $x^2 + 12x + 20 = 0$
5. $x^2 - 4 = 0$
6. $3x^2 + 6x = 0$

4.3 Solve quadratic equations by method of completing squares.

The following steps are followed when solving quadratic equation using the method of completing squares:

Step 1: Convert the equation to the form: $ax^2 + bx + c = 0$ if it is not in this form

Step 2: Divide through by a to get: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Step 3: Move the constant term (term without x) to the right hand side (RHS) of the equation. We obtain

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Step 4: Complete the square in the Left Hand Side (LHS) by adding $(\frac{1}{2} \times \ coefficient\ of\ x)^2$ to both sides to obtain the following equation:
Note: coefficient of x in the equation $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ is $\frac{b}{a}$, we divide it by 2 to get $\frac{b}{2a}$ and then square it to get $\left(\frac{b}{2a}\right)^2$. Now we add this to both sides of the equation $x^2 + \frac{b}{a}x = -\frac{c}{a}$ and we obtain

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

Step 5: The LHS will become $\left(x + \frac{b}{2a}\right)^2$ and the RHS will become

$$-\frac{c}{a} + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\begin{aligned} &= \frac{b^2}{4a^2} - \frac{c}{a} \\ &= \frac{b^2}{4a^2} - \frac{c}{a} \\ &= \frac{b^2 - 4ac}{4a^2} \end{aligned}$$

thus we arive at the equation

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

We will then make x the subject of formula

Step 5: x can now be obtained using

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples:

1. $2x^2 + 3x + 4 = 0$
2. $x^2 + 3x + 2 = 0$
3. $6x^2 + 11x + 4 = 0$
4. $x^2 + 12x + 20 = 0$
5. $x^2 - 4 = 0$
6. $3x^2 + 6x = 0$

4.4 Solve quadratic equations by formula

The following steps are followed when solving quadratic equation using the formula method:

Step 1: Identify the values of a, b and c in the QE: $ax^2 + bx + c = 0$

Step 2: Apply the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

to find the values of x

Examples:

1. $2x^2 + 3x + 4 = 0$
2. $x^2 + 3x + 2 = 0$
3. $6x^2 + 11x + 4 = 0$
4. $x^2 + 12x + 20 = 0$
5. $x^2 - 4 = 0$
6. $3x^2 + 6x = 0$

4.5 Discriminate the roots.

As explained, every quadratic equation has 2 roots. The quantity responsible for this is the Determinant which is defined as

$$D = b^2 - 4ac$$

for the quadratic equation

$$ax^2 + bx + c = 0$$

The following are the different cases of root with respect to the determinants

S/N	Criteria	Roots Description
1	$D > 0$	Distinct Real roots
2	$D = 0$	Repeated Real roots
3	$D < 0$	Imaginary/Complex roots

Examples: Determine the nature of the roots of the following QEs and hence obtain the roots

1. $y^2 - 5y - 14 = 0$
2. $3x^2 + 3x - 18 = 0$
3. $2m^2 - 15m + 27 = 0$
4. $1 + t + 2t^2 = 0$
5. $2y^2 - 3y + 40$

4.6 Form equations whose roots are given in different methods.

We have discussed how to obtain the roots of a QE using different methods. Now we want to see how we can obtain the QE when the roots are given. This can be done in 2 different ways which are discussed in the following sections

4.6.1 Method 1:

Suppose α and β are the roots of a QE, then the QE can be obtained by expanding the following expression

$$(x - \alpha)(x - \beta) = 0$$

Examples: Find the quadratic equations whose roots are

1. 2 and 3
2. -2 and 13
3. $\frac{1}{2}$ and $\frac{5}{3}$
4. 0.2 and 3.1
5. $\frac{2}{3}$ and 3

4.6.2 Method 2:

Suppose α and β are the roots of a QE, then the QE is given by

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Examples: Find the quadratic equations whose roots are

1. 2 and 3
2. -2 and 13
3. $\frac{1}{2}$ and $\frac{5}{3}$
4. 0.2 and 3.1
5. $\frac{2}{3}$ and 3

Chapter 5

Permutation and Combination

5.1 Permutation

Permutation of objects means all possible arrangement of of objects where the order is important. Take for example three letters **A**, **B** and **C**. These letters can be arranged as follows:

A B C
A C B
B A C
B C A
C A B
C B A

The above arrangement was done while putting emphasis on the order of arrangement, this is why **A B C** is one arrangement and **A C B** is another different one. This is called permutation. Therefore, there are 6 different ways of permuting (arranging) three letters.
Now consider the permutation of four letters **A B C** and **D**. This can be done in the following ways

A B C D	**B A C D**	**C A B D**	**D A B C**
A B D C	**B A D C**	**C A D B**	**D A C B**
A C B D	**B C A D**	**C B A D**	**D B A C**
A C D B	**B C D A**	**C B D A**	**D B C A**
A D B C	**B D A C**	**C D A B**	**D C A B**
A D C B	**B D C A**	**C D B A**	**D C B A**

If we continue arrangement in the above manner, we observe the following

S/N	No. of Objects	No. of Ways of Arrangements
1	3	$6 = 3 \times 2 \times 1 = 3!$
2	4	$24 = 4 \times 3 \times 2 \times 1 = 4!$
3	5	$120 = 5 \times 4 \times 3 \times 2 \times 1 = 5!$
4	6	$720 = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6!$

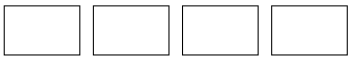
Therefore, the number of ways of arranging n objects is $n!$.

5.1.1 Permutation Formula

The fundamental theorem of counting states that:

> "if a task can be performed in n_1 ways, a second task in n_2 ways and a third task in n_3 ways and so on, then the total number of distinct ways of performing all tasks together is $n_1 \times n_2 \times n_3 \times ...$"

Consider 5 different books say A, B, C, D and E. Suppose we want to arrange 4 out of the five books. To do this, we assume there are 4 spaces for the arrangement, i.e.



We start by putting one of the books in the first box (space), there are 5 different ways doing this since you have 5 books to select from. After choosing a book we are now left with 4 books to choose from. We then move to the second box.
To fill the second box, there are 4 ways of doing it because we have 4 books left to choose from. After choosing a book, we are left with only 3 books. We then move to the third box.
To fill the third box, there are 3 ways of doing it because we have 3 books left to choose from. After choosing a book, we are left with only 2 books. We then move to the fourth box.
Finally, to fill the fourth box, there are 2 ways of doing it because we have 2 books left to choose from. After choosing a book, we now use the fundamental theorem of counting to calculate the number of ways

of arranging 4 books out of the five books, Thus we have,
5 ways of arranging the first book, 4 ways of arranging the second book, 3 ways of arranging the third book and finally 2 ways of arranging the fourth book. Therefore the number of ways of arranging 4 books out of 5 books is given by

$$5 \times 4 \times 3 \times 2 = 120$$

We write ${}^5P_4$ to represent the number of ways of arranging 4 out of 5 objects, hence

$${}^5P_4 = 5 \times 4 \times 3 \times 2 = 120$$

${}^5P_4$ is pronounced "5 permutation 4"
In a similar manner, we have the following,
The number of ways of arranging 4 out of 5 objects is

$${}^5P_4 = 5 \times 4 \times 3 \times 2 = 120$$

The number of ways of arranging 3 out of 5 objects is

$${}^5P_3 = 5 \times 4 \times 3 = 60$$

The number of ways of arranging 2 out of 5 objects is

$${}^5P_2 = 5 \times 4 = 20$$

The number of ways of arranging 5 out of 7 objects is

$${}^7P_5 = 7 \times 6 \times 5 \times 4 \times 3 = 2520$$

and so on.
Now, lets generalize, consider

$${}^7P_3$$

we have

$$\begin{aligned} {}^7P_3 &= 7 \times 6 \times 5 \\ &= 7 \times 6 \times 5 \times \left(\tfrac{4\times3\times2\times1}{4\times3\times2\times1}\right) \\ &= \tfrac{7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1} \\ &= \tfrac{7!}{4!} \\ &= \tfrac{7!}{(7-3)!} \end{aligned}$$

In a similar manner

$$^5P_4 = \frac{5!}{(5-4)!}$$
$$^5P_3 = \frac{5!}{(5-3)!}$$
$$^7P_5 = \frac{7!}{(7-5)!}$$

and so on.

This takes us to the following Permutation theorem.

The theorem states that:

> "The number of ways of arranging r objects out of n objects written as ^nP_r is given by $^nP_r = \frac{n!}{(n-r)!}$"

Proof:

Recall that $^nP_r = n(n-1)(n-2)...(n-(r-1))$

Thus we have

$$\begin{aligned} ^nP_r &= n(n-1)(n-2)...(n-(r-1)) \\ &= n(n-1)(n-2)...(n-(r-1)) \times \frac{(n-r)(n-r-1)(n-r-2)...\times3\times2\times1}{(n-r)(n-r-1)(n-r-2)...\times3\times2\times1} \\ &= \frac{n(n-1)(n-2)...(n-(r-1))(n-r)(n-r-1)(n-r-2)...\times3\times2\times1}{(n-r)(n-r-1)(n-r-2)...\times3\times2\times1} \\ &= \frac{n!}{(n-r)!} \end{aligned}$$

Hence proved.

Examples:

1. Evaluate the following:
   a. $^{10}P_6$
   b. $^8P_4$
   c. $^5P_5$
2. Show that $^nP_r = (n-r+1) \times ^n P_{r-1}$
3. Box 1 contains the letters A, B, C, D, E, F, G whereas Box 2 contains the letters W, X, Y, Z. How many letter codes can be constructed using
   a. 3 letters from box 1 and 2 letters from box 2?
   b. 2 letters from box 1 and 3 letters from box 2?
   c. Why are the above not equal?

4. How many three - letter initials can possibly be formed using only the letters M, N, Q, R, S, and T, if
   a. Repetitions are allowed?
   b. Repetitions are not allowed?
5. In how many ways can 7 people be put in ten seats?

5.1.2 Permutation of Repeated Objects

In the permutation of n objects and within the n objects there are some that are repeated, for example in the word

NECESSARY

The letter **E** is repeated twice so also is the letter **S**, if there are n_1, n_2, n_3, . . . ,n_k repetitions, then the number of ways of arranging the the n objects where there are n_1, n_2, n_3, . . . ,n_k repetitions is given by

$$\frac{n!}{n_1!n_2!n_3!...n_k!}$$

Examples:

1. How many different arrangements can be made using the letters of the following words
   a. MATHEMATICS
   b. HIPPOPOTEMUS
   c. MATHEMATICIANS
   d. MISSISSIPPI
2. A team consists of 5 ND 1 student, 3 ND 2 students and 4 HND 1 students. How many ways can they be arranged on a line if the students of the same class must be together?
3. How many different car numbers can be formed using the 10 letters (A to J) and 8 numbers (0 to 7) if each number must have 3 letters and 3 digits with the letters at the beginning.

4. In how many ways can 3 prizes be won by 12 students of a class
   a. If no student can win more than on prize
   b. If no restriction is placed on the winning of a prize
   c. If only a student wins exactly 3 prizes

5.1.3 Permutation with Restrictions

Supposing we are to arrange n objects in a row such that r objects out of the n objects are to be together is given by the formula

$$[n-(r-1)]!r!$$

and if r objects out of the n objects are not to be together is given by the formula

$$[n-(r-1)](n-r)(n-r)!$$

Examples:

1. **Solution:**

2. **Solution:**

NOTE THAT: $0! = 1! = 1$.

5.1.4 Circular Permutation

Supposing n objects are to be arranged in a ring or circle, then this can be done in $(n-1)!$ ways and if the objects are fixed such that the ring can be turned over, then the required permutations becomes $\frac{(n-1)!}{2}$

The arrangement of n distinct objects in a circle such that r out of the objects are together is given by

$$(n-r)!r!$$

and if the r objects are not together is given by

$$(n-1)[n-(r+1)][n-(r+1)]!$$

Examples:

1. In how many ways can 7 people be seated in a round table?
2. In how many ways can a person arrange 5 bids in a circle?
3. In how many ways can 5 people be arranged in a circle such that two people must sit together?
4. In how many ways can 8 people sit together on a round table so that
   a. 2 people must sit together?
   b. 2 people must not sit together?

5.2 Combination

Combination of objects means all possible selection of of objects. In combination, the order is not important. Take for example four letters **A**, **B**, **C** and **D** and suppose we want to select 3 out of the four letters, then the selection can be made as follows:
The selections

A B C, **A C B**, **B A C**, **B C A**, **C A B** and **C B A**

are all counted as only 1 selection because the order is not important. So therefore, for the selection of 3 out of the 4 letters we have only 4 selections, which are

A B C
A B D
A C D
B C D

In a similar manner, for the selection of 2 out of the 4 letters we have only 6 selections, which are

A B
A C
A D
B C
B D
C D

Also, for the selection of 4 out of 5 letters **A, B, C, D** and **E**, we have only 5 selections, which are

A B C D
A B C E
A B D E
A C D E
B C D E

If we observe from the above, we can deduce the following
Supposing we let $^4C_2$ to represent the number of ways of selecting 2 out of 4 objects (pronounced as 4 combination 2), then we have
$^4C_2 = 6$,
$^4C_3 = 4$ and
$^5C_4 = 5$.
Lets consider the first i. e.

$$^4C_2 = 6$$

We know that

$$^4P_2 = \frac{4!}{(4-2)!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1} = 12$$

combining the above equations we get

$$^4C_2 = 6 = \frac{12}{2} = \frac{^4P_2}{2!} =^4 P_2 \times \frac{1}{2!} = \frac{4!}{(4-2)!2!}$$

Similarly,

$$^4C_3 = 4 = \frac{24}{6} = \frac{^4P_3}{3!} =^4 P_3 \times \frac{1}{3!} = \frac{4!}{(4-3)!3!}$$

Also,

$$^5C_4 = 5 = \frac{120}{24} = \frac{^5P_4}{4!} =^5 P_4 \times \frac{1}{4!} = \frac{5!}{(5-4)!4!}$$

Generally, the number of ways of selecting r objects out of n objects is given by the formula

$$^nC_r = \frac{n!}{(n-r)!r!}$$

Examples:

1. Evaluate the following
   a. $^6C_3$
   b. $^5C_3$
   c. $^8C_2$
   d. $^7C_0$
   e. $^{10}C_{10}$
2. A committee of 6 members (4 males and 2 females) is to be formed from 10 males and 4 females. In how many ways can this be done?
3. A committee of 5 members is to be formed from 3 females and 4 males. In how many ways can it be formed if
   a. at least one female is included as a member?
   b. at least one ,male is included as a member?
4. Prove the following
   a. $^nC_{n-r} =^n C_r$
   b. $^nC_r +^n C_{r+1} =^{n+1} C_{r+1}$
5. In a class of 14 boys and 10 girls, a committee of 7 is to be formed. How many committees are possible if
   a. anybody can serve in the committee?
   b. the committee is to have exactly 4 boys?
   c. the committee is to contain at least 4 boys
6. Simplify the following
   a. $^{10}C_7 +^{10} C_6$
   a. $^7C_{r+1} +^7 C_r$
   a. $^{2r}C_r +^{2r} C_{r-1}$

5.3 Define the basic idea of set theory

5.3.1 Definition of a Set

A set is a well-defined collection or group of objects. For example

A set of all assets of an individual

Set of all courses offered by ND 1 ACC

Set Theory is more manifested in Industries. Example in manufacturing

Employees set

Customers Set

Machines Set

Raw Materials set

Consumables Set

Packing Materials Set.... etc.,

Applied to business operations, set theory can assist in planning and operations. Every element of business can be grouped into at least one set such as accounting, management, operations, production and sales. Within those sets are other sets. In operations, for example, there are sets of warehouse operations, sales operations and administrative operations.

5.3.2 Ways of Describing a Set

There are 3 ways to describ a set, these are:

1. Listing its Elements:
2. Verbal Description:
3. Using Mathematical Inclusion Rule:

5.4 Define subsets, universal, disjoint and non disjoint sets

5.4.1 Subsets

A set S is called a subset of another set A, if A contains all elements of S. This is denoted by

$$S \subset A$$

(Example: in class)

5.4.2 Universal Set

If some sets are under consideration, say set A, B and C, then a universal set U is any set that contains elements of all the sets under consideration. (Example: in class)

5.4.3 Disjoint and Non disjoint Sets

Two sets are said to be disjoint if they do not have a common element, otherwise, they are call non disjoint sets. (Example: in class)

5.4.4 Types of Sets

1. Empty/Null Set
2. Singleton Set: Example: If $A = \{1\}$ and $B = \{a\}$
3. Finite and infinite Sets
4. Union of Sets: Example: If $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$ then $A \cup B = \{1, 2, 3, a, b, c\}$
5. Intersection of Sets: Example: If $A = \{1, 2, 3, b\}$ and $B = \{1, a, b, c, 3\}$ then $A \cap B = \{1, 3, b\}$
6. Difference of Sets
7. Compliment of Set: Example: If $A = \{1, 2, 3\}$ and the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ then $A' = \{4, 5, 6, 7, 8, 9, 10\}$
8. Power Set

5.5 State the laws of set

The following are the Laws of Algebra of Sets

1. Cummutative Laws: For any 2 sets A and B
   a. $A \cup B = B \cup A$
   b. $A \cap B = B \cap A$
2. Associative Laws: For any three finite sets A, B and C;
   a. $(A \cup B) \cup C = A \cup (B \cup C)$
   b. $(A \cap B) \cap C = A \cap (B \cap C)$
3. Distributive Laws: For any three finite sets A, B and C;
   a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4. De Morgans Laws: For any 2 sets A and B
   a. $(A \cup B)' = A' \cap B'$
   b. $(A \cap B)' = A' \cup B'$
5. Idempotent Laws:
   a. $A \cap A = A$
   b. $A \cup A = A$

5.6 Use Venn diagrams to illustrate operations of set.

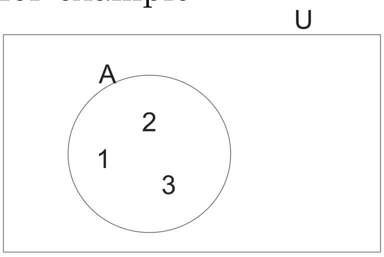
Venn Diagrams (so named after the 18th century English logician, John Venn) are pictorial representation of sets. This basically consists of 2 plane shapes:

1. The Circle: Used to represent an ordinary set (not a universal set), and
2. The Rectangle: Used to represent the Universal set.

The following rule are followed when representing sets on a Venn Diagram:

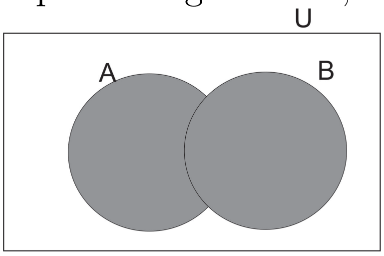
Rule 1: Elements/number of elements of a set is written inside the circle representing it, for example

$A = \{1, 2, 3\}$



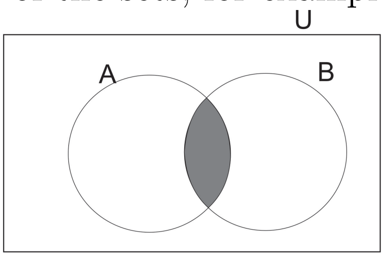
Rule 2: Union of 2 or more sets is shown by shading the inner parts of all the circle representing the sets, for example

$A \cup B$



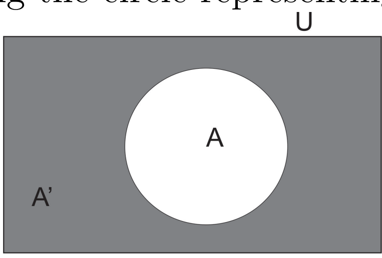
Rule 3: Intersection of 2 or more sets is shown by shading the overlapping portion of the sets, for example

$A \cap B$



Rule 4: Compliment of a set is shown by shading the whole universal set excluding the circle representing the set, for example

A'



5.7 Apply sets in counting

Examples:

1. Two home products were advertised on television at the same time. The Nelson Ratings Company uses boxes attached to television sets to determine what product are actually being bought. In its survey of 1000 homes, it was observed that 153

households bought both products, 736 bought the first product and 55 households did not buy either of the products.

a. How many households bought the second product only?

b. How many households bought the both products?

2. A company has a large of computer assistants, each of whom is competent in the use of at least one of 3 utility packages: Word processor (W) Database Management system (D) and a Spreadsheet (S). A survey shows that 30 can use a word processor, 25 can use a Database Management system and 28 are competent in the use of a Spreadsheet. Of the computer facility assistants who can use a Database Management System, 14 can also use a word processor while 6 have no other skill. 6 of the computer assistants can use a word processor and spreadsheet but not a database system while 4 have all three skills. Determine the number of computer facility assistants who are members of the following sets:

   a. $W \cup D \cup S'$

   b. $(W \cup S)' \cap S$

   c. $D \cap S$

   d. Universal set

3. In a recent survey people were asked if the took a vacation in the summer, winter, or spring in the past year. The results were 73 took a vacation in the summer, 51 took a vacation in the winter, 27 took a vacation in the spring, and 2 had taken no vacation. Also, 10 had taken vacations at all three times, 33 had taken both a summer and a winter vacation, 18 had taken only a winter vacation, and 5 had taken both a summer and spring but not a winter vacation.

   a. How many people had been surveyed?

   b. How many people had taken vacations at exactly two times of the year?

c. How many people had taken vacations during at most one time of the year?

d. What percentage had taken vacations during both summer and winter but not spring?

4. Twenty four people go on holidays. If 15 go swimming, 12 go fishing, and 6 do neither, how many go swimming and fishing? Draw a Venn diagram and fill in the number of people in all four regions

5. A company employed 316 people, 208 of whom are men, 152 including all the women, are business administrators. Find the number of men that are business administrators

6. In a certain class, 220 students take one or more of chemistry, physics and maths. 120 take physics (P), 80 take maths (M) and 70 take chemistry (C). Nobody takes physics and chemistry and 4 students take physics and maths.

   a

   i Using set notation and the letters indicated above, write down the 2 statements in the last sentence.

   ii Draw a Venn diagram to illustrate the information

   b How many students take

   i both chemistry and maths

   ii maths only

7. In preparing a time table for a class of 380 students the following facts were taken into consideration:
   250 students take History
   270 students take French
   280 students take Agric. Science
   200 students take History and French
   230 students take French and Agric. Science
   210 students take History and Agric. Science
   If 180 take all 3 subjects;

a express these in a Venn diagram showing the number of students who offer the 3 subjects

b

i how many students take only one subject?

ii how many students take at least 2 subjects?

iii how many offered none of the 3 subjects?

8. In a technical secondary school, 150 students study carpentry and 170 study blacksmithing. Of these, 60 students study both carpentry and blacksmithing. Find out how many students study either carpentry or blacksmithing or both.

Chapter 6

Understand the properties of arithmetic and geometric progressions

6.1 Define an Arithmetic progression (A.P.)

An arithmetic progression (AP) is a sequence of numbers such that the difference of any two successive members is a constant. For example, the sequence

$$1, 2, 3, 4, ...$$

is an arithmetic progression with common difference 1. In other words an AP is a sequence of numbers that increase or decrease by a number called a common difference. That is to say, the next term in the sequence is obtained by either adding or subtracting the common difference from the immediate term before it. The following are examples of APs

1. $4, 0, 4, 8, ...$, common difference is -4
2. $2, 4, 6, 8, ...$, common difference is 2
3. $1, \frac{1}{2}, 0, -\frac{1}{2}, -1...$, common difference is $-\frac{1}{2}$
4. $200, 400, 600, ...$
5. $80, 78, 76, ...$
6. $10, 8, 6, ...$
7. $4, 11, 18, 25, ...$

8. $22, 17, 12, 7, ...$

9. $5, 0, 5, 10, 15, ...$

NOTE: The common difference can be a positive number or a negative number and to obtain the common difference of an AP just subtract the first number from the second, or the second number from the third, or the third number from the fourth etc.

6.2 Obtain the formula for nth term and the first n terms of an A.P.

6.2.1 Formula for nth term of an AP

The formula for the nth term of an AP is used to obtain any term of the AP. If a is the first term of an AP and d the common difference, then the nth term of the AP is given by

$$n^{th}term = a + (n-1)d$$

For example, consider the AP: $2, 4, 6, 8, ...$, the first term is 2, the second term is 4 and the third term is 6. Thus the common difference is 2 because $4-2 = 6-4 = 2$. We can obtain the fifth term by adding the common difference to the fourth term, i.e. $8 + 2 = 10$. Hence the fifth term is 10, but to obtain the fifth term using the formula we have

$$n^{th}term = a + (n-1)d$$

therefore the fifth term can be obtained as follows NOTE: $a = 2, d = 2$ and $n = 5$ since we need the fifth term

$$\begin{aligned} n^{th}term &= a + (n-1)(d) \\ 5^{th}term &= 2 + (5-1)(2) \\ &= 2 + (4)(2) \\ &= 2 + 8 \\ &= 10 \end{aligned}$$

6.2.2 Formula for The Sum of first n terms of an AP

Suppose the first term of an AP is a and the common difference d, then the sum of the first n terms is given by

$$S_n = \frac{n}{2}[a + (n-1)d]$$

Note that when you add up the terms of a sequence, the result you get is called a series. For example, consider the AP

$$1, 3, 5, 7, ...$$

then the series generated from the AP is

$$1 + 3 + 5 + 7 + ...$$

Examples: Find the sum of the first 15 terms of the following APs

1. $2, 5, 8, ...$
2. $-6, -3, 0, 3, ...$
3. First term of the AP is 5 and common difference is -4
4. The 3^{rd} term is 4 and the 8^{th} term is 36
5. $1, 3, 5, 7, ...$

6.2.3 More examples on AP

Examples:

1. The 1st term of an A.P is 6, the last term is 60, and the sum is 330. Find the number of items in the series and the common difference.
2. The 8th and 12th terms of an A.P are 18 and 26 respectively. Find the 1st term and the common difference.
3. If the sum of the 8th and 9th terms of an A.P is 36 and the 4th term is 12, find the common difference.

4. If 4, p, q, 13 are consecutive terms of an A.P., find the value of

   i p, q

   ii (p + q) x (p q)

5. The 10th term of an A.P is 68 and the common difference is 7, find the 1st term of the sequence.

6.3 Define an Geometric progression (G.P.)

A geometric progression (GP) is a sequence of numbers such that the ratio of any two successive members is a constant. For example, the sequence

$$1, 2, 4, 8, ...$$

is a GP with common ratio 2. In other words a GP is a sequence of numbers that increase or decrease by multiplying a number called a common ratio. That is to say, the next term in the sequence is obtained by multiplying the immediate term before it by the common ratio. The following are examples of GPs

1. $4, 8, 16, 32, ...$, common ratio is 2
2. $2, -4, 8, -16, ...$, common ratio is -2
3. $4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, ...$, common ratio is $\frac{1}{2}$
4. $200, 400, 800, ...$
5. $80, -40, 20, ...$
6. $10, 20, 40, ...$
7. $4, 12, 36, ...$
8. $22, 11, 5.5, ...$
9. $5, 1, -\frac{1}{5}, ...$

NOTE: The common ratio can be a positive number or a negative number and to obtain the common ratio of an GP just divide the 2nd number by the first number, or the third number by the second, or the fourth number by the thrid etc.

6.3.1 Formula for nth term of an GP

The formula for the nth term of a GP is used to obtain any term of the GP. If a is the first term of a GP and r the common ratio, then the nth term of the GP is given by

$$n^{th} term = ar^{n-1}$$

For example, consider the GP: $2, 4, 8, ...$, the first term is 2, the second term is 4 and the third term is 8. Thus the common ratio is 2 because $\frac{8}{4} = \frac{4}{2} = 2$. We can obtain the fourth term by using the multiplying the third term by the common ratio, i.e. $8 \times 2 = 16$. Hence the fourth term is 16, but to obtain the fourth term using the formula we have

$$n^{th} term = ar^{n-1}$$

therefore the fourth term can be obtained as follows NOTE: $a = 2, r = 2$ and $n = 4$:

$$\begin{aligned} n^{th} term &= ar^{n-1} \\ 5^{th} term &= 2 \times 2^{(4-1)} \\ &= 2 \times 2^3 \\ &= 2 \times 8 \\ &= 16 \end{aligned}$$

6.3.2 Formula for The Sum of terms of a GP

Suppose the first term of a GP is a and the common ratio r, then the sum of the first n terms is given by

$$S_n = \frac{a(1 - r^n)}{1 - r}, forr < 1$$

and

$$S_n = \frac{a(r^n - 1)}{r - 1}, forr > 1$$

and also the sum to infinity is given by

$$S_\infty = \frac{a}{1 - r}$$

Examples: Find the sum of the first 15 terms of the following APs

1. If the 1st term of a G.P is 2 and the 4th term is 54, find the:

   a 6th term

   b 10th term/

2. The 3rd term of a G.P is 12 and the 9th term is 768. Find the two possible values of the 6th term.

3. The 4th term of a G.P is 20 and the 7th term is 160. Find the common ratio, the first term and the 12th term.

4. The 5th term of a G.P. is greater than the 4th term by 1312 , and the 4th term is greater than the 3rd term by 9. Find:

   a the common ratio

   b the first term.

5. Find the sum of the 1st 9 terms of the G.P 24, 12, 6, 3, . . .

6. If the 2ndterm of a G.P is 6 and the 5th term is 162, find the 8th term, nth term and the sum of the 1st 6 terms

6.4 Define Arithmetic Mean (AM) and Geometric Mean (G.M.)

6.4.1 Arithmetic Mean (AM)

The Arithmetic mean for a sequence of numbers is obtained by summing up all the terms in the sequence and dividing the result obtained by the number of terms in the sequence. For example, let $\{2, 3, 4, 5, 6\}$ be a sequence of numbers, then the arithmetic mean is obtained as

$$AM = \frac{2+3+4+5+6}{5} = 4$$

6.4.2 Geometric Mean (GM)

The Geometric mean for a sequence of numbers is obtained by multiplying all the terms in the sequence and taking the n^{th} root of the result where n is the total number of terms in the sequence. For example, let $\{2, 3, 4, 5, 6\}$ be a sequence of numbers, then the geometric mean is obtained as

$$GM = \sqrt[5]{(2 \times 3 \times 4 \times 5 \times 6)} = (2 \times 3 \times 4 \times 5 \times 6)^{\frac{1}{5}}$$

6.5 Define convergency and divergence of series.

Consider a series

$$a_1 + a_2 + a_3 + a_4 + a_5 + ...$$

the series is said to be convergent if it approaches some limit. Roughly speaking, an infinite series

$$a_1 + a_2 + a_3 + a_4 + a_5 + ...$$

is said to converge if it adds up to a finite number and diverges if it does not. However, there different of proven tests which can be carried out to check whether an infinite series is convergent or divergent.

Chapter 7

Binomial Theorem of Algebraic Expressions

7.1 Mathematical Induction

Mathematical induction is a special way of proving things like statements, laws, theorems, formula etc. It is mostly used to establish a given statement/formula for all natural numbers. The idea behind Mathematical induction is simple, it follows three steps as follows:

STEP 1: Prove that the statement/formula is true for the first number

STEP 2: Assume it is true for any number k

STEP 3: Then show that it is true for the next number after k i.e. $k+1$

If the above steps are proved, then that simply means that it is true for all natural numbers.

Examples: Prove the following using Mathematical Induction

1. $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+...+\frac{1}{n(n+1)}=\frac{n}{n+1}$
2. 3^n-1 is even
3. $5+10+15+...+5n=\frac{5n(n+1)}{2}$
4. 3^n-2n-1 is divisible by 4
5. $1+3+5+..+(2n-1)=n^2$
6. $4^{2n}-1$ is divisible by 3

7. $\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+...+\frac{n}{2^n}=2-\frac{n+2}{2^n}$

8. $1^2+2^2+3^2+...+n^2=\frac{1}{6}n(n+1)(2n+1)$

7.2 The Pascal Triangle

The Pascal triangle Method is an easy way for solving a binomial expansion with a small index. A binomial is an algebraic expression of the sum or the difference of two terms for example $(a+b),(2+x),(y+x)$, etc.

When a binomial has an index (power), there is a need to expand it to get the actual expression for example, consider the binomial

$$(a+b)^2$$

This can be expanded as follows:

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+ab+ba+b^2\\ &= a^2+ab+ab+b^2 \text{ since } ba=ab\\ &= a^2+2ab+b^2\end{aligned}$$

In a similar expansion for $(a+b)^3$we have

$$\begin{aligned}(a+b)^3 &= (a+b)(a+b)(a+b)\\ &= (a^2+2ab+b^2)(a+b)\\ &= a^3+a^2b+2a^2b+2ab^2+b^2a+b^3\\ &= a^3+3a^2b+3ab^2+b^3\end{aligned}$$

Expanding in this way upto the index 5, we obtain the following table

$$\begin{aligned}(a+b)^1 &= a+b\\ (a+b)^2 &= a^2+2ab+b^2\\ (a+b)^3 &= a^3+3a^2b+3ab^2+b^3\\ (a+b)^4 &= a^4+4a^3b+6a^2b^2+4ab^3+b^4\\ (a+b)^5 &= a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\end{aligned}$$

From the table above we can observe the following

1. The powers of a is decreasing by 1 across the terms

2. The powers of b is increasing by 1 across the terms

3. There is a special pattern made by the coefficients of the terms in the expansion as follows:

Binomial		Coefficients
$(a+b)^1$	$=$	1 1
$(a+b)^2$	$=$	1 2 1
$(a+b)^3$	$=$	1 3 3 1
$(a+b)^4$	$=$	1 4 6 4 1
$(a+b)^5$	$=$	1 5 10 10 5 1

The coefficient in the above table forms a Triangular pattern known as the Pascal Triangle. In the Pascal triangle, coefficients of a particular power can be found using the coefficient of the previous power. The following table illustrates how the Pascal triangle is generates

1 1

$\searrow$ add $\swarrow$

1 2 1

$\searrow$ add $\swarrow$ $\searrow$ add $\swarrow$

1 3 3 1

$\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$

$\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$

$\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$

Supposing you are asked to expand the binomial $(x+2)^5$ using the Pascal Triangle methods, the following steps should be followed,

Step 1: Bring out the coefficients for the power 5 using the Pascal triangle and these are the following coefficients

$$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$$

Step 2: Arrange the coefficients in the following format

$$1 \quad +5 \quad +10 \quad +10 \quad +5 \quad +1$$

Step 3: Pick the first term in the binomial and multiply each coefficient with it starting from the first coefficient with the power and as you move across the terms in the expansion reduce one from the power as follows (note that the first term in the binomial expression is x)

$$1\times x^5 \quad +5\times x^4 \quad +10\times x^3 \quad +10\times x^2 \quad +5\times x^1 \quad +1\times x^0$$

Step 4: Now pick the second term in the binomial and multiply it across the terms but in this case starting with power zero and adding 1 to the power when multiplying across the terms of the expansion as follows (note that the second term the binomial expression is +2)

$$1\times x^5\times 2^0+5\times x^4\times 2^1+10\times x^3\times 2^2+10\times x^2\times 2^3+5\times x^1\times 2^4+1\times x^0\times 2^5$$

Step 5: Simplify the expression, thus we have

$$\begin{aligned}(x+2)^5 &= x^5\times 1+5x^4\times 2+10x^3\times 4+10x^2\times 8+5x\times 16+1\times 32\\ &= x^5+10x^4+40x^3+80x^2+80x+32\end{aligned}$$

NOTE: The following are worthy to note when expanding a binomial expression using the Pascal Triangle

- When a term in a binomial has a negative sign for example $(x-2)^3, (y-3)^4, (4-x)^7$ etc, then when picking the terms for the expansion, the negative sign must be used i.e. in the binomial $(x-3)^4$, the first term is x while the second term is -3 not 3.
- When you are asked to expand a binomial so that one of the terms should be in ascending powers or descending powers, then you will have to rearrange if the need arise. For example in the expansion of $(a+b)^4$ we obtain

  $$(a+b)^4=(1)a^4(b^0)+(4)a^3(b^1)+(6)a^2(b^2)+(4)a^1(b^3)+(1)a^0(b^4)$$

  we observe that the power of a is decreasing while the power of b is increasing, therefore the expansion is in descending powers of a and also ascending powers of b.
  Another way we can express the binomial $(a+b)^4$ is $(b+a)^4$ since

  $$(a+b)^4=(b+a)^4$$

  Now expanding $(b+a)^4$ we get

  $$(b+a)^4=(1)b^4(a^0)+(4)b^3(a^1)+(6)b^2(a^2)+(4)b^1(a^3)+(1)b^0(a^4)$$

  the expansion is now in descending powers of b and ascending powers of a. So the position of the term is responsible for either ascending or descending power of the term in the expansion. You can always interchange the positions of the terms to suit the question. Some of these are done below

$$(a-b)^5 = (-b+a)^5$$
$$(2+x)^4 = (x+2)^6$$
$$(7-y)^8 = (-y+7)^8$$
$$(-3-b)^5 = (-b-3)^5$$

etc.

- The Pascal triangle method is mostly used if the power of the binomial is less than 10. If the power exceeds 10, then it becomes difficult for someone to use this method because bringing out the coefficients may be very difficult for example if you are given $(a+5)^{33}$ to expand. Bringing out the triangle of to the power 33 may pose alot of problems. A more accommodating method for binomial expansion with a large index is the Binomial Theorem.

Examples: Use the Pascal Triangle method to expand the following binomials

1. $(3-2x)^5$
2. $(\frac{x}{2}-5)^7$
3. $(x+3)^6$ in ascending powers of x
4. $(5-3x)^9$ in descending powers of x
5. $(2-\frac{1}{x})^5$

7.3 Binomial Theorem for Positive Integer Indices

Binomial theorem for positive integer indices is a simplified way of expanding binomial express especially the ones with large indices like $(a+b)^{59}$ etc. The theorem was formed using a special way of obtaining

the coefficients. This special way is explained below.
Let us consider the coefficients of the binomial expression with powers 3, 4 and 5 i.e.

Binomial		Coefficients										
$(a+b)^3$	$=$				1		3		3		1	
$(a+b)^4$	$=$			1		4		6		4		1
$(a+b)^5$	$=$	1	5		10			10		5		1

Now observe the following

Binomial		Coefficients					
$(a+b)^3$	:	$1={}^3C_0$	$3={}^3C_1$	$3={}^3C_2$	$1={}^3C_3$		
$(a+b)^4$	:	$1={}^4C_0$	$4={}^4C_1$	$6={}^4C_2$	$4={}^4C_3$	$1={}^4C_4$	
$(a+b)^5$	:	$1={}^5C_0$	$5={}^5C_1$	$10={}^5C_2$	$10={}^5C_3$	$5={}^5C_4$	$1={}^5C_5$

Therefore using this idea, the coefficients for any power can be obtained without the Pascal Triangle. For example, the coefficients of the power 6 will be

$$ {}^6C_0,\ {}^6C_1,\ {}^6C_2,\ {}^6C_3,\ {}^6C_4,\ {}^6C_5, \text{ and } {}^6C_6 $$

Generalizing we get the Binomial theorem for positive integer indices as follows

Theorem: (Binomial Theorem for Positive Integer Indices): The theorem states that

$$ (a+b)^n = \sum_{r=0}^{r=n} {}^nC_r a^{n-r} b^r $$

where n is a positive integer.

Proof: We are going to use Mathematical Induction to prove this theorem

Step 1:

Test for $n = 1$ to see whether

$$ (a+b)^n = \sum_{r=0}^{r=n} {}^nC_r a^{n-r} b^r $$

thus we have the left hand side (LHS) of the equation to be

$$ (a+b)^1 = a + b $$

and the right hand side (RHS) is

$$\begin{aligned}\textstyle\sum_{r=0}^{r=1} {}^{1}C_r a^{1-r} b^r &= {}^{1}C_0 a^{1-0} b^0 +^{1} C_1 a^{1-1} b^1 \\ &= \text{1(a)(1)+1(1)(b)} \\ &= \text{a+b}\end{aligned}$$

Therefore the LHS $= (a+b)^n = \sum_{r=0}^{r=n} {}^{n}C_r a^{n-r} b^r =$ RHS, hence it is true for $n = 1$

Step 2:

Assuming it is true for $n = k$ where k is an integer, then

$$(a+b)^k = \sum_{r=0}^{r=k} {}^{k}C_r a^{k-r} b^r$$

Step 3: For $n = k + 1$ we have

$$\begin{aligned}(a+b)^{k+1} &= (a+b)^k \cdot (a+b) \\ &= \textstyle\sum_{r=0}^{r=k} {}^{k}C_r a^{k-r} b^r \cdot (a+b) \\ &= (a+b) \cdot \textstyle\sum_{r=0}^{r=k} {}^{k}C_r a^{k-r} b^r\end{aligned}$$

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Therefore, supposing we want to expand $(a+b)^4$ we first of all bring out the coefficient just as we did in the Pascal triangle method but in this case, we are going to use the binomial coefficient method as follows; the coefficients for the expansion of $(a+b)^4$ are

$${}^{4}C_0, \quad {}^{4}C_1, \quad {}^{4}C_2, \quad {}^{4}C_3, \quad {}^{4}C_4$$

the remaining process is the same as that of Pascal triangle method, thus we have the following

$$(a+b)^4 = {}^{4}C_0 a^4 b^0 + {}^{4}C_1 a^3 b^1 + {}^{4}C_2 a^2 b^2 + {}^{4}C_3 a^1 b^3 + {}^{4}C_4 a^0 b^4$$

but we know that

$${}^{4}C_0 = 1, \quad {}^{4}C_1 = 4, \quad {}^{4}C_2 = 6, \quad {}^{4}C_3 = 4, \quad {}^{4}C_4 = 1$$

thus

$$(a+b)^4 = a^4 + 4a^3 b^1 + 6a^2 b^2 + 4a^1 b^3 + b^4$$

Similarly for the expansion of $(a+b)^5$, we have the following coefficients

$${}^{5}C_0, \quad {}^{5}C_1, \quad {}^{5}C_2, \quad {}^{5}C_3, \quad {}^{5}C_4, \quad {}^{5}C_5$$

therefore

$$(a+b)^5 = {}^5C_0a^5b^0 + {}^5C_1a^4b^1 + {}^5C_2a^3b^2 + {}^5C_3a^2b^3 + {}^5C_4a^1b^4 + {}^5C_5a^0b^5$$

but

$${}^5C_0 = 1, \quad {}^5C_1 = 5, \quad {}^5C_2 = 10, \quad {}^5C_3 = 10, \quad {}^5C_4 = 5, \quad {}^5C_5 = 1$$

thus

$$(a+b)^5 = a^5 + 5a^4b^1 + 10a^3b^2 + 103a^2b^3 + 5a^1b^4 + b^5$$

Properties of Binomial Expansion

The following are some properties of binomial expansion $(a+b)^n$ where n is a positive integer

1. The number of terms in the expansion is (n+1) which is one more than the index.
2. Every term in the expansion of $(a+b)^n$ can be written in a compact form as
$${}^nC_ra^{n-r}b^r$$
depending on the value of r.
If $r = 0$, we obtain the first term i.e. ${}^nC_0a^{n-0}b^0 = 1 \cdot a^n \cdot 1 = a^n$
If $r = 1$, we obtain the second term i.e. ${}^nC_1a^{n-1}b^1 = n \cdot a^{n-1} \cdot b = na^{n-1}b$
and so on.
3. The first and the last terms are a^n and b^n respectively.
4. The sum of the coefficients is
$${}^nC_0 + {}^nC_1{}^nC_2 + ... + {}^nC_n = 2^n$$
5. ${}^nC_0 = {}^nC_n$ and ${}^nC_1 = n$
6. Progressing from the first term to the last, the exponent of a decreases by 1 from term to term while the exponent of b increases by 1. In addition, the sum of the exponents of a and b in each term is n.

Examples: Use the Binomial theorem to find the first 6 terms of the following binomials

1. $(x^2 - \frac{1}{x})^2 0$
2. $(\frac{x}{2} - \frac{y}{3})^1 5$ in ascending powers of x
3. $(2 - \frac{x}{2})^1 2$
4. $(a - 5)(a + 3)^1 0$
5. $(x + 2y^2)^1 2$

7.3.1 Finding the Constant term (independent of x) in an Expansion

Consider the following expansion

$$(x + 2)^2 = x^2 + 4x + 4$$

The term 4 in the expansion is called the constant term because there is no x attached to it. It is possible to obtain the constant term in an expansion without expanding the binomial. This is done using the compact form of the binomial $(a + b)^n$ which is

$${}^nC_r a^{n-r} b^r$$

. To get the constant term, we look for the value of r (in the compact form of a binomial expression) that will make the power of x be 0. We then substitute it and evaluate, for example, to find the constant term in the expansion of

$$(x + 6)^5$$

, we first of all find the compact form which is

$${}^nC_r a^{n-r} b^r$$

but comparing $(x + 6)^5$ with $(a + b)^n$, we get $a = x$, $b = 6$ and $n = 5$, thus we have the compact form as

$${}^nC_r a^{n-r} b^r = {}^5C_r x^{5-r} 6^r$$

We can see that the power of x in the compact form is $5 - r$ and the value of r that will make this power 0 is $r = 5$, we then substitute it to obtain the constant term as

$$^5C_5x^{5-5}6^5 = 1 \cdot x^0 \cdot 6^5 = 6^5$$

Another example is finding the constant term of the expansion of

$$(x - \frac{2}{x})^4$$

In this case we first of all obtain the compact form, we have $a = x$, $b = \frac{-2}{x}$ and $n = 4$, thus the compact form is

$$^nC_ra^{n-r}b^r = {}^4C_rx^{4-r}(\frac{-2}{x})^r$$

If you observe, we cannot get the power of x since there are 2 xs in the compact form, so what we will do is to use the law of indices to try to bring the xs together before taking the power, thus we have

$$\begin{aligned} ^nC_ra^{n-r}b^r &= {}^4C_rx^{4-r}(\tfrac{-2}{x})^r \\ &= {}^4C_rx^{4-r}\tfrac{(-2)^r}{(x)^r} \\ &= {}^4C_r\tfrac{x^{4-r}}{(x)^r}(-2)^r \\ &= {}^4C_rx^{4-r-r}(-2)^r \\ &= {}^4C_rx^{4-2r}(-2)^r \end{aligned}$$

Now we have the power of x as $4 - 2r$, so to get the value of r that will make this power 0, we equate the power to 0 and find the value of r, thus we have

$$\begin{aligned} 4 - 2r &= 0 \\ \Rightarrow \quad -2r &= -4 \\ \Rightarrow \quad r &= \tfrac{-4}{-2} \\ \Rightarrow \quad r &= 2 \end{aligned}$$

we then substitute the value of $r = -2$ to obtain the constant term of the expansion, thus

$$\begin{aligned} ^4C_rx^{4-r}(\tfrac{-2}{x})^r &= {}^4C_rx^{4-2r}(-2)^r \\ &= {}^4C_2x^{4-2(2)}(-2)^2, \text{ substituting } r = 2 \\ &= 6 \cdot x^0 \cdot 4 \\ &= 6 \cdot 4 \\ &= 24 \end{aligned}$$

Thus the constant term is 24

Examples: Find the constant term (term independent of x) in the expansion of each of the following binomial

1. $(x^2 + \frac{3}{x})^9$
2. $(\frac{1}{x} - 2x)^12$
3. $(5x - \frac{x^2}{2})^6$
4. $(2x - \frac{8}{x^2})^5$
5. $(\frac{1}{x} - \frac{2}{x})^9$

7.3.2 Finding the Coefficient of a Particular term in an Expansion

Finding the coefficient of a term in an expansion is similar to finding the constant term, the difference is that instead of finding r that will making the power of x to be 0, we find the value of r that will make the power of x to be the power of x in the required term. For example, consider the expansion of $(x + 2)^4$ is

$$(x + 2)^4 = x^4 + 8x^3 + 24x^2 + 24x + 16$$

The constant term is 16
The coefficient of x in the expansion is 24
The coefficient of x^2 in the expansion is 24
The coefficient of x^3 in the expansion is 8 and
The coefficient of x^4 in the expansion is 1
The coefficients can be obtained using the compact form, for instance if we want to get the coefficient of x^3 in the expansion of $(x + 2)^4$, we first of all find the compact form then find the value of r that will make the power of x to be 3. We then substitute it and find the coefficient. Hence we have the compact form as

$${}^4C_r x^{4-r}(2)^r$$

since we want the coefficient of x^3, we will look for the value of r that will make the power of x in the compact form (i.e. $4 - r$) be equal to 3. Clearly, $r = 1$ will make $4 - r$ to be 3, thus we substitute

$$\begin{aligned} {}^4C_r x^{4-r}(2)^r &= {}^4C_1 x^{4-1}(2)^1, \text{ substituting } r = 1 \\ &= 4 \cdot x^3 \cdot 2 \\ &= 8x^3 \end{aligned}$$

Therefore the coefficient of x^3 in the expansion of $(x + 2)^4$ is 8.

Examples:

1. Find the coefficient of x^2 and x^6 in the expansion of $(\frac{1}{x} - 2x)^{10}$
2. Calculate the coefficient of x^4 in the expansion of $(x^2 - \frac{2}{x})^5$
3. Determine the coefficient of x^{-2} in the expansion of $(x - \frac{1}{x})^4$
4. Find the term in x^9y^6 in the expansion of $(x + 2y^2)^12$
5. Find the coefficient of x^2 in the expansion of $(\frac{x^2}{4} - 2x)^9$

7.4 Binomial Theorem for Negative and Fractional Indices

Supposing we are asked to use the binomial theorem to expand expressions such as

$$\frac{1}{(x+2)^4} = (x+2)^{-4}$$
$$\sqrt{(2+x)} = (2+x)^{\frac{1}{2}}$$
$$\sqrt[3]{(x-1)} = (x-1)^{\frac{1}{3}}$$

then, this is not going to be possible using the binomial theorem for positive integer indices because, if we take $(x + 2)^{-4}$ for example, we will have to bring out the coefficients which are

$${}^{-4}C_0, {}^{-4}C_1, {}^{-4}C_2, ...$$

but in evaluating ${}^{-4}C_0$, we will have to find $(-4)!$ which does not exist since we can only find the factorial of a positive number.
This poses a problem, that is, how to expand a binomial with a negation or fractional indices. To solve this problem, the binomial theorem for positive integer indices was modified to accomodate the negative and fraction indices. This modification is illustrated as follows.
The binomial theorem for positive integer indices is

$$(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b^1 + {}^nC_2a^{n-2}b^2 + {}^nC_3a^{n-3}b^3 + ... + {}^nC_nb^n$$

The main constraint that deos not allow us to use this theorem for negative or fractional index is the fact that we can not find the factorial of a negative number or a fraction therefore the combination cannot be found. What we will do is to find a way to find ${}^nC_r$ without using factorial and this is found below

${}^nC_0 = 1$ this is a known fact that any number combination 0 is 1 (can be proven)

${}^nC_1 = n$ this is also a known fact that any number combination 1 is that number (can be proven)

${}^nC_2 = \frac{n!}{(n-2)!\cdot 2!} = \frac{n(n-1)(n-2)!}{(n-2)!\cdot 2!} = \frac{n(n-1)}{2!}$

${}^nC_3 = \frac{n!}{(n-3)!\cdot 3!} = \frac{n(n-1)(n-2)(n-3)!}{(n-3)!\cdot 3!} = \frac{n(n-1)(n-2)}{3!}$

${}^nC_4 = \frac{n!}{(n-4)!\cdot 4!} = \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!\cdot 4!} = \frac{n(n-1)(n-2)(n-4)}{4!}$

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${}^nC_n = 1$ this is a known fact that any number combination itself is 1 (can be proven)

Replacing the above in the binomial theorem for positive integer indices we get the binomial theorem for negative or fractional indices as follows

$$(a+b)^n = a^n + na^{n-1}b^1 + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + ... + b^n$$

1. Write down the first three terms of the expansion of $\frac{1}{(1=X)^2}$
2. Find the first 4 terms of the power series expansion of $(1-\frac{x}{2})^{-\frac{1}{2}}$
3. Expand $(8+3x)^{\frac{1}{3}}$ in ascending powers of x as far as the term in x^2.
4. Obtain the first 4 terms of the expansion of $\sqrt{x^2-5}$
5. Find the first 5 terms of the expansion of $\frac{1}{(x-3)^5}$

7.5 Application of Binomial Theorem to Approximation

Expression like $(1+x)^n$ can be approximated by using the first few terms for the expansion if x is sufficiently small because higher powers of x can be negligible. Therefore if x is sufficiently small, the expansion of $(a+x)^n$ canhave the following approximations

$$\text{Linear approximation } (a+x)^n \approx a^n + na^{n-1}x$$
$$\text{Quadratic approximation } (a+x)^n \approx a^n + na^{n-1}x + \tfrac{n(n-1)}{2!}a^{n-2}x^2$$

Examples:

1. For example, write down the first 4 terms of the expansion of $(1-x)^5$ in ascending powers of x. Use your answer to find $(0.998)^5$ correct to 3 decimal places.

   Solution:

   Expanding $(1-x)^5$ we have
   $$\begin{aligned}(1-x)^5 &\approx {}^5C_0 1^5 + {}^5C_1 1^4(-x) + {}^5C_2 1^3(-x)^2 + {}^5C_3 1^2(-x)^3\\ &= 1 - 5x + 10x^2 - 10x^3\end{aligned}$$
   To approximate $(0.998)^5$ using the expansion, compare it with $(1-x)^5$ and find the value of x, thus we have
   $$\begin{aligned}(0.998)^5 &= (1-x)^5\\ \Rightarrow \quad 0.998 &= 1-x\\ \Rightarrow \quad x &= 1-0.998\\ \Rightarrow \quad x &= 0.002\end{aligned}$$
   therefore we have $x = 0.002$, hence

   $(1-x)^5 \approx 1-5x+10x^2-10x^3$ substituting the value of x we get
   $$\begin{aligned}(0.998)^5 &= (1-0.002)^5 \approx 1-5(0.002)+10(0.002)^2-10(0.002)^3\\ &= 1-0.01+0.00004-0.00000008\\ &= 0.99003992\\ &= 0.990 \text{ to 3 d.p.}\end{aligned}$$

2. Find the first four terms of the expansion of $(1+\frac{x}{2})^{10}$ in ascending powers of x. Hence, find the value of (1.002)610 correct to 5 d.p.

3. Using the first 3 terms of the binomial series for $(1+x)^{\frac{1}{3}}$, find $\sqrt[3]{0.99}$ correct to 3 d.p.

4. Find the first 3 terms of the expansion of $(1-x)^{-\frac{1}{2}}$ and use it

to calculate $\frac{1}{\sqrt{1.01}}$ correct to 3 d.p.

5. Without using tables or calculator, find the value of $(2+\sqrt{5})^4 + (2-\sqrt{5})$

Chapter 8

Understand the basic concepts and manipulation of vectors and their applications to the solution of engineering problems.

8.1 State the definitions and representations of vectors.

8.1.1 Definition of a Vector

A vector is a quantity that has magnitude as well as direction examples include: distance, speed, velocity, acceleration, force, mass, momentum, energy, work, power etc. A vector is described by both magnitude and direction.

8.1.2 Representation of Vectors

A vector can be represented graphically as a directed line segment, i.e. a portion of a line on which direction is indicated. See figure below

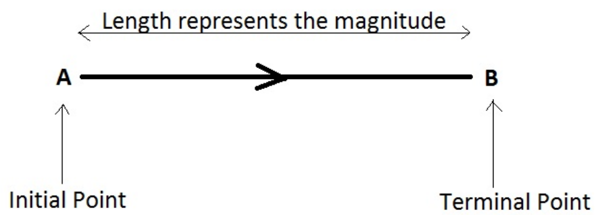

Fig: Directed line segment

Point A is the initial point while point B is the terminal point of the vector. The length of the line represents the magnitude of the vector

while the direction is indicated on the line. The symbolic notation for the vector is $\vec{AB}$ while the symbol for its magnitude is $|\vec{AB}|$. Example of vectors include the following
$\vec{AB} = 2i + 5j + 4k$
$\vec{AB} = 15i - 5j + 10k$
$\vec{AB} = 3i - 3j - 12k$
$\vec{AB} = 90i - 2k$
Note that each of the vectors above has 3 components namely, the i, j and k components. For the vector $\vec{AB} = 2i + 5j + 4k$, 2 is the i component, 5 the j component and 4 the k component.

8.1.3 Define a position vector.

Let P be a point with coordinate (a, b, c) and O be the origin i.e. the point $(0, 0, 0)$, then the vector $\vec{OP}$ is called a position vector of the point P and it is obtained as follows

$$\begin{aligned} \vec{OP} &= (a-0)i + (b-0)j + (c-0)k \\ &= ai + bj + ck \end{aligned}$$

8.2 Define unit vector

A unit vector is a vector whose magnitude is 1. The magnitude of a vector $\vec{AB} = ai + bj + ck$ is given by

$$|\vec{AB}| = \sqrt{a^2 + b^2 + c^2}$$

9.4 Explain scalar multiple of a vector 9.5 List the characteristics of parallel vectors 9.6 Identify quantities that may be classified as vector e.g. displacement velocity, acceleration, force etc. 9.7 Compute the modulus of any given vector up to 2 and 3 dimensions. 9.8 State the parallelogram law in solving problems including addition and subtraction of vectors 9.9 Apply the parallelogram law in solving problems including addition and subtraction of vectors. 9.10 Explain the concept of components of a vector and the meaning of orthogonal components. 9.11 Resolve a vector into its orthogonal components. 9.12 List characteristics of coplanar localized vectors. 9.13 Define the resultant or composition of coplanar vectors. 9.14 Compute the resultant of coplanar forces acting at a point using algebraic and graphical methods. 9.15 Apply the techniques of resolution and resultant to the solution of problems involving coplanar forces. 9.16 Apply vectoral techniques in solving problems involving relative velocity. 9.17 State the scalar product of two vectors. 9.18 Compute the scalar product of given vectors. 9.19 Define the cross product of the vector product or two vectors. 9.20 Calculate the direction ratios of given vectors. 9.21 Calculate the angle between two vectors using the scalar product.

Chapter 9

Understand the concept of equations and methods of solving different types of equations and apply same to engineering problems.

9.1

10.1 Explain the concept of equation, ie. A = B where A and B are expressions. 10.2 List different types of equations:- Linear, quadratic, cubic, etc. 10.3 State examples of linear simultaneous equations with two unknowns and simultaneous equations with at least one quadratic equation. 10.4 Apply algebraic and graphical methods in solving two simultaneous equations involving a linear equation and a quadratic equation. 10.5 Apply the algebraic and graphical methods in solving two simultaneous quadratic equations. 10.6 Define a determinant of nth order. 10.7 Apply determinants of order 2 and 3 in solving simultaneous linear equations.

Chapter 10

Understand the definition, manipulation and application of trigonometric functions.

10.1 Define the basic trigonometric ratios

sine, cosine and tangent of an angle. 11.2 Derive the other trigonometric ratios; cosecant, secant and cotangent using the basic trigonometric ratios in 11.1 above. 11.3 Derive identities involving the trigonometric ratios of the form; Cos2+Sin2 = 1, Sec2 = 1 + tan2, etc. 11.4 Derive the compound angle formulae for sin (A+B), Cos (A+B) and Tan (A+B).

Examples:

1.

2.

3.

4.

5.