EDO UNIVERSITY IYAMHO

Department of Computer Science

MTH 111 Algebra and Trigonometry

Instructor: *Alhassan Charity Jumai*, email: charity.alhassan@edouniversity.edu.ng
Lectures: Tuesday, 8am – 10.00 am, LT1, phone: (+234) 80682325043
Office hours: Wednesday, 10:00 to 12.00 PM

General overview of lecture: The course introduces some fundamental concepts in Real number system, Including, Set theory and notations, Binary operations and its properties: closure, distributive, associative, commutative laws with examples, Real sequences and series. Elementary ratios of convergence of arithmetic, geometric and other simple series, Theory of quadratic equations, Polynomials and partial fractions

Prerequisite: The students are expected to have a strong background in the fundamentals of Simple inequalities, the principle of mathematical induction, Addition and factor formulae. Complex numbers, Algebra of complex numbers, the Argand diagram, Permutation and Combination and the Binominal theorem.

Learning outcomes: At the completion of this course, students are expected to:

i. to better understand Real number systems,
ii. to understand the concept of sets theory and its notation,
iii. to understand Binary operations, its properties
iv. to understand the notion of Real sequences and series.
v. To understand elementary ratios of convergence of arithmetic, geometric and other simple series,
vi. To solve problems on quadratic equations, Polynomials and partial fractions.
vii. To understand simple inequalities,
viii. To solve the principle of mathematical induction,
ix. To solve problems on Complex numbers, Algebra of complex numbers, the Argand diagram,
x. To understand the term Permutation and Combination.
xi. To solve problems on Binominal theoremn

Assignments: We expect to have 5 individual homework assignments throughout the course in addition to a Mid-Term Test and a Final Exam. Home works are due at the beginning of the class on the due date. Home works are organized and structured as preparation for the midterm and final exam, and are meant to be a studying material for both exams. There will also be 3 individual programming projects in this class. The goal of these projects is to have the students experiment with very practical aspects of compiler construction and program analysis.

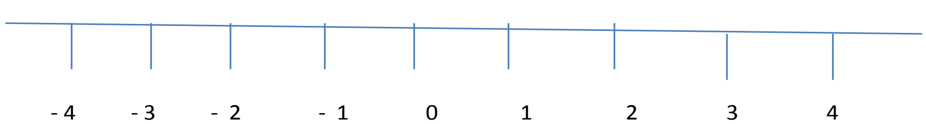
Grading: We will assign 10% of this class grade to home works, 10% for the programming projects, 10% for the mid-term test and 70% for the final exam. The Final exam is comprehensive.

Textbook: The recommended textbook for this class
Title: *Introduction to Basic Mathematics*
Authors: Timmy Obiwuru
Reprint 2003,2005,2007 2nd Edition
Printed and bound by Panaf Press
Panaf villa
Plot 196/197
Dawaki Rockville
Behind news engineering company
Gwarinpa,Abuja.
08023400164, 08023244739

Main Lecture: Below is a description of the contents. We may change the order to accommodate the materials you need for the projects.

REAL NUMBER SYSTEM

One of the most important properties of the real numbers is that points on a straight line that can represent them. As in Fig 1, we choose a point, called the origin, to represent 0 and another point, usually to the right, to represent 1.Then there is a natural way to pair off the points on the line and the real numbers, that is, each point will represent a unique real number and each real number will be represented by a unique point. We refer to this line as the *real line.* Accordingly, we can use the words point and number interchangeably. Those numbers to the right of 0, i.e. on the same side as 1, are called the *positive numbers* and those numbers to the left of 0 are called the *negative numbers.* The number 0 itself is neither positive nor negative.



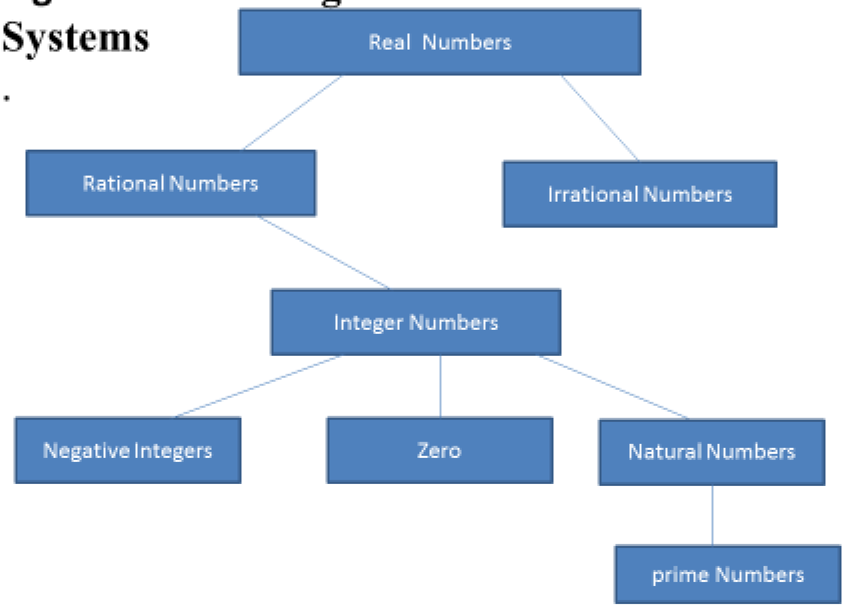
The integers are those real numbers…, -3, -2, -1, 0, 1, 2, 3,…

We denote the integers by Z; hence we can write Z = { …,-3, -2, - 1, 0, 1, 2,3,…} The integers are also referred to as the "whole numbers''. One important property of the integers is that they are "closed" under the operations of addition, multiplication and subtraction; that is, the sum, product and difference of two integers is again an integer. Notice that the quotient of two integers, e.g. 3 and 7, need not be an integer; hence the integers are not closed under the operation of division.

The *rational numbers* are those real numbers, which can be expressed as the ratio of two integers. Notice that each integer is also a rational number since, for example, 5 = 5/1; . The rational numbers are closed not only under the operations of addition, multiplication and subtraction but also under the operation of division (except by 0). In other words, the sum, product, difference and quotient (except by 0) of two rational numbers is again a rational number.

The irrational numbers are those real numbers which are not rational, that is, the set of irrational numbers is the complement of the set of rational numbers Q in the real numbers R; hence Q' denote the irrational numbers. Examples of irrational numbers are $\sqrt{3}$, π, $\sqrt{2}$ etc

Figure 2 Line Diagram of the Real Number Systems



SET THEORY

A set is a collection of well defined object. The objects are called elements or members of the set.

Examples

The set of students reading engineering in Edo University, Edo State.

The set of tools used by a carpenter

The set of all even numbers.

There are basically two methods of describing a set.namely:

The Roster method, and

The Rule method

The Roster Method

If the elements of a set are enumerated and enclosed in braces with comma separating one element from another, the set is said to have been described using the roster method.

i) A ={ a, b, c, d}

ii) B= {1,3, 5,7…}

iii) C={ Jane, Usman, John}

A is a set consisting of four elements a, b, c and d. We conventionally denote a set using capital letter and its elements using small letters. At times numbers could be the elements of a set. For example. B is the set of all odd numbers

The Rule Method

Instead writing a set by listing its elements in braces as in the case of roster method, a set can also be written by enclosing in braces a mathematical statement of their common property as follows:

Example 1 D=$\{x \middle| x\ is\ a\ month\ beginning\ with\ J\}$

This is the rule method of describing a set and it is read as ‘’D is the set of all x such that each x is a month beginning with J’’. The slash ‘’|’’ is read such that’.At times the colon ‘:’ may be used in place of the slash ‘’|’’.

D={January,June,July} in roster method

Example 2. E={x|(x-2)(x+7)=0},by rule method. In this case E is the set of all x that satisfy the equation (x-2)(x+7)=0.

$\therefore E = \{2, -7\}, by\ roster\ method.$

iii) F={X|x is an integer,$0 < x < 8\}, by\ rule\ method. F\ is\ the\ set\ \ of\ all\ x\ that\ are\ integer$ and between 0 and 8 not inclusive.

Types of set

Null or set Empty sets

If a set does not have any element it is called a null or empty set. A null or empty set is denoted by$\emptyset\ or\{\}$ i.e $\emptyset = \{\}$

Example boys having three eyes in our class.

Equivalent sets

Two sets A and B are said to be equivalent if each element of A has a one – one correspondence with element of B and vice versa. The set of the names of students in a class and the set of their matriculation numbers can be said to be equivalent. The set of points on the number line and the set of all real numbers are also examples of equivalent sets.

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Finite and Infinite Set

A set is finite if it’s elements or members can be counted. A set is infinite if the counting does not terminate.

Equal Set

Two sets A and B are said to be equal if A is a subset of B and B is also a subset of A. It should be noted that it does not matter if a particular element is repeated. A = (4,5,7), B=(4,5,7)

Complement set

If A is a set, the complement of set A denoted by A' is the set which consists of elements in the universal set U that are not in the set A. Using the rule method. A’ can be written as

A’={x|x∈ $to\ A, x \in to\ U$}

Venn Diagram

In sets we at times use diagrams to illustrate certain concepts clearer at a glance than the use of symbolic manipulations. Diagrams in sets try to explain at a glance the relationship existing among the elements of the universal set and one or more of its subsets. A venn diagram consists of a rectangle or square representing the universal set with its subsets represented by circles and enclosed in the rectangle. Each of the following is an example of a venn diagram.

Operations on set

1) The Intersection of sets

If there are two sets A and B, the intersection of two sets denoted by A∩B is the set that contains all the elements that are in both A and B. A∩B is read as 'A intersection B'. The symbol ∩ is called 'intersection'.

A∩B= $\{x|x \in A \; and \; x \in B\}$ by the rule method.

If A= {all those in the football team} and B={all those in the hockey team}then A∩B is the set of all those in the football and in the hockey teams.

$x \in A \; and \; x \in B \; \leftrightarrow x \in$ (A ∩ B)

Note the explanation of the following symbols.

$a \rightarrow b \; read \; 'a \; implies \; b'$.And it means that b must follow from a and a may not necessary follow from b.

$a \leftrightarrow b$ reads 'a implies b and b implies a' or

A if and if b'. if means b follows from a and vice versa

Example Let the universal set U=$\{x|1 \leq x \leq 20, x \; is \; an \; integer\}$

P=$\{x$-1|10$\leq x \leq 18, x \; is \; even\}$

Q=$\{x|2 < x \leq 15, x \; is \; odd\} and$

R=$\{x|1 < x < 20, x \; is \; a \; multiple \; of \; 3\}$

Determine the following sets using the roster method.

a) P∩ Q (b) Q∩ R (c) P∩ R (d) P∩ Q' (e) (P∩ $Q)'$

(f) (P'∩ $Q)'$ (g) P ∩ (Q∩ R) (h) (P ∩ Q) ∩ R

SOLUTON

We first list the sets, P,Q, R and U using the roster method as follows:

U= {1,2,3,4…20}

P= {9,11,13,15,17}

P'={1,2,3,4,5,6,7,8,10,12,14,16,18,19,20}

Q= {3,5,7,9,11,13,15}

Q'={1,2,4,6,8,10,12,14,16,17,18,19,20}

R={3,6,9,12,15,18}

R'={1,2,4,5,7,8,10,11,13,14,16,17,19,20}

Hence the sets are specified below:

a) P∩ Q ={9,11,13,15}

b) $Q\cap R = \{3,9,15\}$

c) $P\cap R = \{9,15\}$

d) $P\cap Q' = \{17\}$

e) (P∩ Q) is the set in (a).Hence

(P∩ Q)'

={1,2,3,4,5,6,7,8,10,12,14,16,17,18,19,20}

f)=P'∩ Q={3,5,7}

∴ (P'∩Q)={1,2,4,6,8,9,10,11,12,13,14,15,16,17,18,19,20}

Union of sets

The union of two sets A and B denoted by $A \cup B$ and read A' union B' is another set that contains elements that are either in A or B or both. Symbolically,

$A \cup B = \{x|x \in A \text{ or } x \in B \text{ or } x \in (A \cup B)\}$ by rule method.

$x \in (A \cup B) \leftrightarrow x \in A \text{ or } x \in B$

In the union set, the elements in the intersection are included only once.

Generally, In systematically listing the elements of the union set, say $A \cup B$. write all the elements of

The first set A and secondly include from set B its elements that are not in set A and vice versa.The result is the set $\cup B$.

Example

Given that U={a,b,c,d,e,f,g}

P={a,b,d,f}

Q={a,c,d,g} and

R={f,c,a,d}

Determine the following sets by the roster method.

a) P ∪ Q' (b) P' ∪ R' (c) (Q-R') ∪ P (d) R∩ $(P - Q)$

e)) P∩ (Q ∪ R) (f) Q' ∪ (P ∪ R) (g) P ∪ (Q ∪ R)

Solution

a) P ∪ Q'={a,b,d,f,e}

b) P' ∪ R' ={c,e,g,b}

(c) (Q-R') ∪ P

(Q-R') ={a,c,d,g} since R'={b,e,g}

(Q-R') ={a,c,d}

(Q-R') ∪ $P = \{a, c, d\}$ ∪{a,b,d,f}

(Q-R') ∪ $P = \{a, c, d, b, f\}$

d) R ∩ (P-Q') ={f}

BINARY OPERATIONS

Definition of Binary operation

A binary operation on a set K is a rule called the law of composition such as addition and multiplication by which we combine two numbers (or elements).For example, if a, b$\in K$ and a$* b = c, then\ *$ $is\ the\ binary\ operation$ which combines elements a and b to produce c. The word 'binary' arises as a result of our combining two numbers or elements. The four arithmetic operations +,-,x, and ÷ $are\ the\ most\ familiar\ binary\ operations$

Many other symbols such as

$\Delta, \sim, \circ, \nabla,$

e.t c. can be used as binary operations in place of $*$.The following examples will help us to understand better the concept of binary operation.

Example 1

The operation $*$ is defined by the relation p$* q = \frac{(p-q)^2}{p+q}$ $on\ the\ set\ of\ real\ numbers$

Evaluate (a) 2$*$ 5 (b)5 $*$2 (c) 3 $*$7 (d) 7 $*$ 3

2$*$ 5=$\frac{(2-5)^2}{2+5}$=$\frac{9}{7}$

5$*$ 2=$\frac{(5-2)^2}{5+2}$=$\frac{9}{7}$

2$*$ 5= 5$*$ 2

c) 3$*$ 7 $= \frac{(3-7)^2}{3+7}$=$\frac{16}{10}$ =$\frac{8}{5}$

d) 7$*$ 3 $= \frac{(7-3)^2}{7+3}$=$\frac{16}{10}$ =$\frac{8}{5}$

3$*$ 7= 7$*$ 3

In general p$* q$=q$* p$ for all real numbers p and q

Example 2

The operation Δ is defined by the relation

r $\Delta s = |r - s|$ on the set of real numbers

Evaluate a) 1 Δ 6 (b)6 Δ 1 (c) 2 Δ 9 (d)9 Δ 2

Where $|r - s|$ means the absolute value of the difference between r and s

a) 1 Δ 6 $=|1-6|=|-5|=5$

b) 6 Δ 1 $=|6-1|=|5|=5$

(c) 2 Δ 9 $=|2-9|=|-7|=7$

d) 9 Δ 2 $=|9-2|=|7|=7$

Solve for

1) the operation $* \, is \, defined \, by \, the \, relation$

$u^*v = u^2 + v + 2$ on the set of real numbers where u$\neq 0$ $and \, v \neq 0$

Evaluate

a) 2^*4 $(b)4^*2$ (c) 0^*3 (d) 3^*0

Question 2

The operation $\sim$ is defined by the relation

$p{\sim}q=\frac{p-q}{pq}$ on the set of real numbers where p$\neq q$,p$\neq 0, q \neq 0$ evaluate

a) 2~6 b) 6~2 c) 8~1 d) 1~8

Closure of a set

Let k be a non empty set. If $\forall \, a, b \in k, a * b \in$ $k \, then \, the \, set \, k \, is \, said \, to \, be \, closed \, under \, the \, binary$

Operation $*$.if the set is not closed under a given binary operation$*$ then we need to give at least one case whereby a^*b $\in$k for a,b $\in k$.

Example investigate whether or not the set k= {-1, 0,1} is closed under the operations +,-,x and ÷

+	**-1**	**0**	**1**
-1	-2	-1	0
0	-1	0	1
1	0	1	2

Laws of binary operation

Commutative binary operations

A binary operation on a set k is said to be commutative if $\forall\ a, b \in k,\ a \quad b = b \quad a$.The binary operation defined in examples above are commutative

The operation + and x are commutative because a+b=b +a $\forall a, b \in R\ and\ ab = ba\ \forall a, b \in R. But\ the\ operations\ -\ and$ ÷ are not commutative because a-b≠ $b - a \forall a, b \in R$(except a=b)

a ÷ b ≠ b ÷ a $\forall a, b \in R$ (except a=b)

Associative Binary operations

Let the set k be closed under the binary operation $\Delta, if\ \forall\ a, b \in k$

(a Δ b) Δ c = a Δ(b Δ c)

Then we say that the operation Δ is associative.

The operations + and x are associative because

(a +b)+c=a+(b +c)

and (ab)c=a(bc)

However, operations- and ÷ are not associative because

SEQUENCE AND SERIES

A sequence is a set of element in some definite order or a string of objects, like numbers, that follow a particular pattern. The individual elements in a sequence are called terms. For example

3,6,9,12,15,18, 21

Pattern: add 3 to the previous number to get the next number

0,12,24,36,48,60,72

Pattern: add 12 to the previous number to get the next number

Let us represent every term in a sequence by a variable. let us call the nth term in a sequence Xn the first term is represented by $X_1, The\ second\ term\ X_2 \ldots$

Example

If X_n=n-2^n

Find X_1,X_2 and X_3

Solution

$X_n = n - 2^n$
n=1
$X_1 = 1 - 2^1$
∴ X_1= -1

If $X_n = n - 2^n$
n=2
$X_2 = 2 - 2^2$
$X_2 = 2 - 4$
$X_2 = -2$
$X_n = n - 2^n$
n=3
$X_3 = 3 - 2^3$
$X_3 = 3 - 8$
$X_3 = -5$
Questions 2
In the sequence $X_{1\,,}X_{2\,,}X_3 \ldots\ X_1 = 1, X_{n+1} = X_n + 8n\ where\ n \geq$
$1.$ Write $X_2\ , X_3, X_4, X_5$.
Hence find the formula for Xn.

$X_1 = 1 = 1^2$
$X_{n+1} = X_n + 8n$
n=1
$X_{1+1} = X_1 + 8 \times 1$
$X_2 = 1 + 8$
$X_2 = 9 = 3^2$
$X_{n+1} = X_n + 8n$
n=2
$X_{2+1} = X_2 + 8 \times 2$
$X_3 = 9 + 16 = 25 = 5^2$
$X_{n+1} = X_n + 8n$

$$X_{3+1} = X_3 + 8(3)$$
$$X_4 = 25 + 24 = 49 = 7^2$$

This is the squares of consecutive odd numbers from ONE thus

$$X_n = (2n - 1)^2$$

ARITHMETIC PROGRESSION (A.P)

When a sequence has a constant difference between successive terms it is called an **arithmetic progression** (often abbreviated to AP).

Examples include:
(i) 1, 4, 7, 10, 13, . . . where the **common difference** is 3 and
(ii) *a*, *a*+*d*, *a*+2*d*, *a*+3*d*, . . . where the common difference is *d*.
If the first term of an AP is '*a*' and the common difference is '*d*' then
the *n*'th term is: $T_n = \boldsymbol{a + (n - 1)d}$
Certain symbols are used
a=first term
d=common difference
n=number of terms
$S_n = sum\ of\ the\ first\ n\ terms$

Question 1

What is the 11th term of the sequence 4.3, 3.7, 3. 1, 2.5…

Solution

a=4.3
d=second-first term
d=3.7-4.3
d=-0.6
11th term =a+10d
=4.3+10(-0.6)
=4.3-6
=-1.7

Question 2

If the 2nd and the 7th terms of an A.P. is 25 and the fifth term is 15, find the common difference

Solution

2nd term=a+d
7th term=a+6d
Sum of the 2nd and 7th terms=25
$\therefore (a+d)+(a+6d)$=25

a+d+a+6d=25

2a+7d=25…………………i

Fifth term =a+4d

a+4d=15

a=15-4d…………………..ii

Substituting equation (ii) in (i)

2a+7d=25

2(15-4d)+7d=25

30-8d+7d=25

-d=25-30

-d=-5

d=5

But a=15-4d

a=15-20

a=-5

3)if -8,m,n,19 are in arithmetic progression, find (m,n)

Solution

a=-8

Counting to 19 gives fourth term

Fourth term=19

a+3d=19

-8+3d=19

3d=19+8

3d=27,d=9

But m=a+d(second term)

=-8+9

m=1

n=a+2d(third term)

n=-8+2(9)

n=-8+18

n=10

$\therefore (m,n) = (1,10)$

3) the first term a’ of an A.P is equal to twice the common difference d’, find the terms of d, the 5th term of the A.P

Solution

First term=a

Common difference=d

a=2d

5th term=a+4d

But a=2d by substitution

=2d+4d=6d

Sum of the nth term of an A.P

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

EXAMPLES

An A.P has its first term as 11 and fourth term as 32.Find the sum of the first nine terms.

Solution

a=11

Fourth term =32

a+3d=32 but a=11

11+3d=32

3d=32-11

3d=21

d=$\frac{21}{3}$=7

The sum of the 9th term is

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

EXERCISES

1) The first term of an A.P is 3 and the fifth term is 9.Find the number of terms in the progression if the sum of the progression is 81.

2 The sum of five terms of an A.P is 10 and the sum of their squares is 380.Find the first term and the common difference.

3 If the sum of the 8th and 9th terms of an A.P is 72 and the 4th term is -6.Find the common difference.

ARITHMETIC MEAN

If the numbers p, q, r are three consecutive terms of an A. p. Then q is called the arithmetic mean of p and r. Since p, q, r are consecutive terms of an A.P .It implies that:

q-p=r-q=d

q-p=r-q

2q=p + r or

$q = \frac{p+r}{2}$

That is the arithmetic mean of two numbers is the average of the two numbers.

Example

Find the 15th term and the sum of the first 26 terms of the series 21,17,13,9,5,… Calculate the arithmetic mean of -15 and -23

Solution

$U_n = a + (n-1)d.$

a=21 and d=17-21=-4

$U_{15} = 21 + 14(-4) = -35$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{26} = \frac{26}{2}((42 + 25(-4))$=-754

arithmetic mean=$\frac{-15+(-23)}{2}$=-19

GEOMETRIC PROGRESSION

A series that is of the form a +ar + ar^2+ar^3+…

Is called a geometric progression. Where a is the first term and r is the common ratio. The nth term is $T_n = ar^{n-1}$

Prove that in a G.p

$S_n = \frac{a(1-r^n)}{1-r}$ where $r < 1$

Proof

$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

$rS_n = ar + ar^2 + \ldots + ar^n$

Equation 1 and 2

$S_n - rS_n = a - ar^n$

$(1-r)S_n = a - ar^n$

$\therefore S_n = \frac{a(1-r^n)}{1-r}$. where $r < 1$

This completes the required proof

and

$S_n = \frac{a(r^n-1)}{r-1}$ where $r > 1$

Examples

If 7 and 189 are the first and fourth terms of a Geometric progression respectively, find the three terms of the progression.

Solution

a=7

Fourth term=189

$ar^3 = 189\ but\ a = 7$

$7r^3$=189

r^3=$\frac{189}{7}$

r^3=27

$r^3 = 3^3$

r=3

a=7,r=3,n=3

$S_n = \frac{a(r^n-1)}{r-1}$ where $r > 1$

$S_3 = \frac{7(3^3-1)}{3-1}$

$S_3 = \frac{7(27-1)}{3-1}$

$S_3 = \frac{7(26)}{2}$

$S_3 = 7 * 13 = 91$

2) the sum of the first three terms of a geometrical progression is 38 and the fourth term exceeds the first by 19.find the values of the first term and the common ratio

Solution

For the sum of only three terms to be 38 means $r > 1$ $n = 3$

$S_n = \frac{a(r^n-1)}{r-1}$ where $r > 1$

$= \frac{a(r^3-1)}{r-1}$=38

Fourth term=ar^3

First term=a

Fourth term-First term=19

ar^3-a=19

$a(r^3 - 1)$=19

r^3-1=$\frac{19}{a}$ ii

But $\frac{a(r^3-1)}{r-1}$=38 i

Substituting equation ii in I the values of r^3-1

a($\frac{19}{a} * \frac{1}{r-1}$)=38

$\frac{19}{r-1}$=38

By cross multiplication

19=38(r-1)

19=38r-38

38r=38+19

38r=57

r=$\frac{57}{38}$=$\frac{3}{2}$

But r^3-1=$\frac{19}{a}$

($\frac{3}{2}$)^3-1=$\frac{19}{a}$

$\frac{27-8}{8}$=$\frac{19}{a}$

$\frac{19}{8}$=$\frac{19}{a}$

19a=8$*$ 19

a=$\frac{8*19}{19}$= 8

First term =a=8

Common ratio=r=$\frac{3}{2}$

) Find the sum of the first 8 terms of the series

$\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \ldots$

4) If the 7th and 10th terms of a G.P are 320 and 2560 respectively, find the series.

Solution

3) a= $\frac{1}{27}, r = \frac{1}{9} \times \frac{27}{1} = 3$

$S_n = \frac{a(1-r^n)}{1-r}, \therefore S_8 = \frac{\frac{1}{27}(1-3^8)}{1-3}$=121.5

4) $U_7 = ar^6$ and $U_{10} = ar^9$ $\frac{ar^9}{ar^6}$=$\frac{2560}{320}$=$r^3 =$

$8, r =$ 2

$ar^6 = 320,\ 1.e\ a = \frac{320}{64}$ =5

EXERCISES

1) In a Geometric progression, the first term is 153 and the sixth term is $\frac{17}{27}$ .The sum of the first four terms is?
2) Given that x-2,x-1 and 3x-5 are three consecutive terms of a geometrical progression. Find the positive values of x and the common ratio of the progression
3) 4,x,y, 32 are in G.P Find x, y and the eleventh term of the progression.
4) For what values of x does the series

- $\frac{x}{1+x} + \frac{x}{(1+x)^2} + \frac{x}{(1+x)^3}$+...
- Converge. And to what sum does it then converge

Geometric Mean

If the number a, b,c are three consecutive terms of a G.P., then b is called the geometric mean of a and c. Since a, b,c are consecutive terms of a G.p it follows that

$\frac{b}{a}$=$\frac{c}{b}$=r

$\frac{b}{a} = \frac{c}{b} = b^2 = ac \text{ or } b = \sqrt{ac}$

That is the geometric mean of two numbers is the square root of their product.

Find the geometric mean of 2 and 18.

Convergent series

Consider the geometric series 1+ $\frac{2}{3}+\frac{4}{9}$+$\frac{8}{27}$+…Thus the sum of the first four terms is 2$\frac{11}{27}$.From the above results, we can see that as we add more and more terms, the sum of the series gets nearer to 3.This means that S_n tends to 3 as n tends to infinity. Therefore the limit of S_n as n tends to infinity is 3 and this is written as: $\lim_{n\to\infty} S_n$=3

We say that the series is convergent to sum 3 or sum to infinity of the series is 3.Convergent rule is that -1$< r < 1$ where r is the common ratio

Example

S_n=$\frac{a(1-r^n)}{1-r}$ where $r < 1$

As ‘n’ tends to infinity, the sum becomes

S_∞=$\frac{a(1-r^\infty)}{1-r}$

But when a positive number that is less than one is raised to infinity, the result is Zero

S_∞=$\frac{a(1-0)}{1-r}$

S_∞=$\frac{a}{1-r}$

Question 1

If the sum of the progression 1+x+x^2+…to infinity is 3.Find the value of x

Solution

1+x+x^2+…thus a=1

r=$\frac{second\ term}{first\ term}$=$\frac{x}{1}$

r=x

S_∞=$\frac{1}{1-r}$

S_∞=$\frac{1}{1-x}$

but S_∞=3… given

$\frac{3}{1}$=$\frac{1}{1-x}$

By cross multiplication

3(1-x)=1

3-3x=1

-3x=1-3

-3x=-2

$x=\frac{2}{3}$

Question 2

For what values of x does the series

$\frac{x}{1+x}+\frac{x}{(1+x)^2}+\frac{x}{(1+x)^3}+\ldots$

Converge. And to what sum does it then converge

Solution

For convergent series $-1<r<1$

r=$\frac{second\ term}{first\ term}$=$\frac{x}{(1+x)^2}\div\frac{x}{1+x}$

=$\frac{x}{(1+x)(1+x)}*\frac{1+x}{x}$

r=$\frac{1}{1+x}$

For convergent series.

-1 $<\frac{1}{1+x}<1$

This simplifies to $-1<\frac{1}{1+x}$… (i)

$\frac{1}{1+x}<1$… (ii)

Solving equation (i)

Multiplying both sides by $-(1+x)^2\ for\ absolute\ values$

-$(1+x)^2<\frac{1}{1+x}*\frac{(1+x)^2}{1}$

-$(1+2x+x^2)$ <1+x

$-1-2x-x^2-1<0$

$-x^2-3x-2<0$

Multiplying by minus, the sign of inequalities changes

$x^2-3x-2>0$

(x+2)(x+1) >0

Bring the constant out

-2,-1

x <-2 or x >-1…i

Solving the equation ii

$\frac{1}{1+x} < 1$

Multiplying by $(1+x)^2$

$\frac{1}{1+x}(1+x)^2 < 1(1+x)^2$

$$(1+x) < 1 + 2x + x^2$$

Rearranging the values, the sign of inequality changes.

$1 + 2x + x^2 >$1+x

x^2 +2x-x+1-1 >0

x^2 +x >0

x(x+1) >0

Bring the constant out x> $0 \; or \; x < -1$

Theory of Quadratic Equation

QUADRATIC EQUATIONS

A quadratic equation is an equation whose general form is given as:

$ax^2 + bx + c = 0$

a≠ 0 because the highest power of x is 2 in a quadratic equation.

Note that a,b and c are constants with a as the coefficient of x^2, b is the coefficient of x and c is the constant term.The expression $y = ax^2 + bx + c$, (a≠ 0) is a quadratic function which is a polynomial of degree two.

Methods of quadratic equation

Solution by factors

Examples

$x^2 + 5x - 14$ *can be factorized into* $(x+7)(x-2)$. *The equation* $x^2 + 5x - 14=0$ can be written in the form

(x+7)(x-2)=0 and this equation is satisfied if either factor has a zero value.∴

x+7=0 or x-2=0 and this equation is satisfied if either factor has a zero value.

i.e x=-7 or x=2 are the solutions of the given equation x^2+5x-14=0

By way of revision, then you can solve the following equations with no trouble:

a) x^2-9x+18=0
b) $x^2 + 11$x+28=0
c) $x^2 + 5$x-24=0
d) x^2-4x-21=0

Not all quadratic expressions can be factorized as two simple linear factors. Remember that the test for the availability of factors with $ax^2 + bx + c$ is to calculate whether $(b^2 - 4ac)$ $is\ a\ perfect\ square$

The test should always be applied at the beginning to see whether, in fact, simple linear factors exist.

For example, with $x^2 + 8\mathrm{x} + 15$, $(b^2 - 4ac)$ =8^2-4$\times 1 \times 15 = 64 - 60 = 4$

$x^2 + 8\mathrm{x} + 15$, can be written as the product of two simple linear factors. i.e 4=2^2

But with $x^2 + 8\mathrm{x} + 20,\ \mathrm{a} = 1, \mathrm{b} = 8, \mathrm{c} = 20$

And $(b^2 - 4ac) = 8^2 - 4 \times 1 \times 20 = 64 - 80 =$
$-16, which\ is\ not\ a\ perfect\ square$.

So $x^2 + 8\mathrm{x} + 20$ cannot be written as the product of two simple linear factors

Solution by completing the square method We have seen that some quadratic equations are incapable of being factorized into two simple factors. In such cases, another method of solution must be employed. The following example will show the procedure.

Solve $x^2 - 6x - 4$=0 a=1,b=-6,c=-4

$(b^2 - 4ac) = 6^2 - 4 \times 1 \times (-4)$ =52.Not a perfect square.

$\therefore No\ simple\ factors$

$x^2 - 6x - 4$=0

Add 4 to both sides $x^2 - 6x$ = 4

Add to each side the square of half the coefficient of x:

$x^2 - 6x + (-3)^2$=4+$(-3)^2$

$x^2 - 6x + 9$=4+9

$x^2 - 6x + 9$=13

$(x - 3)^2 = 13$

$x-3=\pm\sqrt{13}$

$x=3+\sqrt{13}$ or $x=3-\sqrt{13}$

$\therefore x=6.606\ or\ x=-0.606$

Solvex^2+8x+5=0

$2x^2+10x-7$=0

$4x^2-16x-3$=0

Solution by formula

We can establish a formula for the solution of the general equation $ax^2+bx+c=0$ which is based on the method of completing the square:

$ax^2+bx+c=0$

Dividing throughout by the coefficient of x, i.e. a

$x^2+\frac{b}{a}$x+$\frac{c}{a}$=0 subtracting $\frac{c}{a}$ from both sides gives

$x^2+\frac{b}{a}$x=-$\frac{c}{a}$ we then add to each side the square of half the coefficient of x:

$x^2+\frac{b}{a}$x+($\frac{b}{2a}$)^2==-$\frac{c}{a}$+($\frac{b}{2a}$)^2

$x^2+\frac{b}{a}$x+$\frac{b^2}{4a^2}$=$\frac{b^2}{4a^2}-\frac{c}{a}$

(x+$\frac{b}{2a}$)^2=$\frac{b^2-4ac}{4a^2}$

$\therefore x+\frac{b}{2a}$=$\pm\sqrt{\frac{b^2-4ac}{4a^2}}$=$\pm\frac{\sqrt{b^2-4ac}}{2a}$

$\therefore x$=-$\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

If $ax^2+bx+c=0$, x=$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Solve the equation using quadratic formula

$x^2-3x-4=0$

a=2,b=-3,c=-4 and , x=$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

, x=$\frac{3\pm\sqrt{(3)^2-4\times2\times(-4)}}{4}$=$\frac{3\pm\sqrt{9+32}}{4}$=$\frac{3\pm\sqrt{41}}{4}$=$\frac{3\pm6.403}{4}$

=$\frac{3+6.403}{4}$ or$\frac{3-6.403}{4}$

x=-0.851 or x=2.351

As an exercise,use the formula method to solve the following

a) $5x^2$+12x+3=0

b)) $3x^2$-10x+4=0

c)) $6x^2 - 8x - 9 = 0$

Solution of cubic equations having at least one linear factor in the algebraic expression.

Example 1

To solve the cubic equation:

$2x^3 - 11x^2 + 18x - 8 = 0$

The first step is to find a linear factor of the cubic expression:

$f(x) = 2x^3 - 11x^2 + 18x - 8$

By application of the remainder theorem. To facilitate the calculation, we first write $f(x)$ in nested form

$f(x)$=[(2x-11)x+18]x-8

Now we seek a value for x,(x=k) which gives a zero remainder on division by (x-k).We therefore evaluate f(1),f(-1),f(2)… etc.

F(1)=1 $\therefore (x-1) is\ not\ a\ factor\ of\ f(x)$

F(-1)=-39 $\therefore (x+1) is\ not\ a\ factor\ of\ f(x)$

F(2)=0 $\therefore (x-2) is\ not\ a\ factor\ of\ f(x)$

We therefore divide f(x) by $(x-2)$ to determining the remaining factor, which is

$2x^2 - 7x + 4$

$2x^2 - 7x + 4$

X-2 $\sqrt{2x^3 - 11x^2 + 18x - 8 = 0}$

$2x^3 - 4x^2$

-$7x^2$+18x

$-7x^2 + 14x$

4x-8

4x-8

0 0

$f(x)$=(x-2) $(2x^2 - 7x + 4)$ and the cubic equation is now written: (x-2) $(2x^2 - 7x + 4)$=0

Which gives x-2=0 or $2x^2 - 7x + 4 = 0$

X=2 and the quadratic equation can be solved in the usual way giving

$2x^2 - 7x + 4 = 0 \therefore \frac{7 \pm \sqrt{49-32}}{4} = \frac{7 \pm \sqrt{17}}{4} = \frac{7 \pm 4.1231}{4}$

x=0.719,x=2.781

EXERCISE

Solve the equation $3x^3 + 12x^2 + 13x + 4 = 0$

SIMPLE INEQUALITIES

You certainly have earlier been exposed to the following symbols

$< which\ is\ read'lessthan'$

$\leq which\ is\ read'less\ than'\ or\ equal\ to'$

$>\ which\ is\ read\ 'greater\ than'$

$\geq which\ is\ read'greaterthan'\ or\ equal\ to'$

Each of these symbols is called an inequality

Examples of inequality are common mathematics

a$< b$,

$5x - 3 > 7$

$\frac{7}{x} \geq 9$ and $x^2 + x \leq 6$

Rules of inequalities

1) if any value x is added to(or subtracted from) both sides of an inequality, the sign of the inequality remains the same. If a$< b, then\ a + x < b + x$

E.g given -4< 5,then -4+2$< 5 + 2, i.e - 2 < 7$

Also, if a$< b,\ then\ a - x < b - x$

2) if any positive value x multiplies or divides both sides of an inequality, the inequality sign remains unchanged. If a<b,then -7$\times 5 < 4 \times 5\ i.e\ 35 < 20$

3) if any negative value x multiplies or divides both sides of an inequality, the inequality sign is reversed. If -6< 30 i.e $\frac{-6}{-3} > \frac{30}{-3}$

Note that the same holds when $>, \geq or \leq$ is used in place of $<$

Example

Find the solution of each of the following and illustrate the solution using a number line.

a) 4x+3≤ -17

b) $\frac{2x-1}{3} < 7$

c) $\frac{24}{x} > 4$

d)$\frac{3}{2x}$ >($\frac{1}{3x} - \frac{1}{6}$)

Solutions

$x \leq \frac{-20}{4}$ i.e x≤ -5

This solution is an infinite interval with closed right hand.

R.H.S inequality:$\frac{2x-1}{3} < 7$,i.e 2x-1< 21 i.e x< 11

L.H.S inequality::$\frac{2x-1}{3} \geq -5\ i.e\ 2x-1 \geq -15\ i.e\ x \geq -7$

The complete solution is -7$\leq x \leq 11$

The interval is finite with a closed left hand.

c) $\frac{24}{x}$ >4

i) if x> 0,then 24$> 4x$

$x < 6$

x> 0

0 <0,then 24 $< 4x$

x> 6

x< 0

The complete solution is 0<×< 6

(x-6)(x-9) < 0.If the product of two brackets has to be negative, then either

1) (x-6) < 0 and (x-9) > 0.i.e x <6 and x > 9.There is no solution.

Or ii) (x-6) > 0 and (x-9) < 0 i.e x $>$ 6 and x < 9.The solution is 6 <x <9.

Find all real numbers x which satisfy the inequality

$$\frac{1}{3}(x+1)-1>\frac{1}{5}(x+4)$$

Solution

$\frac{1}{3}(x+1)-1>\frac{1}{5}(x+4)$

Opening the bracket

$\frac{x}{3}+\frac{1}{3}$-1> $\frac{x}{5}+\frac{4}{5}$

Collecting like terms

$\frac{x}{3}-\frac{x}{5}>\frac{4}{5}+\frac{1}{1}\ -\frac{1}{3}$

$\frac{5x-3x}{15}>\frac{12+15-5}{15}$

$\frac{2x}{15}>\frac{22}{15}$

$\therefore 2x>22$

$x>11$

Question

List all integers satisfying the inequality

-2≤ $2x-6<4$

Gives two equations

-2≤ $2x-6$…….i

2x-6< $4 \ldots \ldots .. ii$

Solving equation (i)

-2 ≤ $2x-6$

2x≥ $6-2$

2x≥4

x≥ 2

Solving equation (ii)

2x-6< 4

2x< $6+4$

2x< 10

x< 5

Partial fraction

(1) if the degree of the numerator of the given fraction is equal to or greater than that of the denominator, divide the numerator by the denominator until a remainder is obtained which is of lower degree than the denominator

2) to every linear factor (ax+b) in the denominator, there corresponds a partial fraction of the form $\frac{A}{(ax+b)}$ $for\ instance$

$\frac{2x+3}{(2x+1)(x+8)} \equiv \frac{A}{2x+1} + \frac{B}{2x+1}$

3) to every repeated factor like $(ax + b)^2$in the denominator,there corresponds two partial fractions of the form A/(ax+$b)^2$ and B/(ax+b).Similarly for factors like

(ax+b)^3 $there\ are\ \ three\ partial\ fractions$

$\frac{A}{(ax+b)^3}, \frac{B}{(ax+b)^2}, \frac{C}{(ax+b)^1}$ $and\ so\ on. E.g$

$\frac{2x+3}{(x+5)^2(x+3)} = \frac{A}{(x+5)^2} + \frac{B}{x+5} + \frac{B}{x+3}$

Express in partial fraction

$\frac{1}{x^2+5x+6}$

Solution

$\frac{1}{x^2+5x+6}$ when factorized is $\frac{1}{(x+3)(x+2)}$

$\frac{1}{x^2+5x+6} \equiv \frac{A}{x+3} + \frac{B}{x+2}$

L.C.M of the Right Hand Side

$\frac{1}{x^2+5x+6} \equiv \frac{A(x+2)+B(x+3)}{(x+3)(x+2)}$

The denominator of the L.H.S =denominator of the R.H.S Equating the numerators

$A(x + 2) + B(x + 3) = 1$

Let $x = -2.$

$That\ is\ what\ value\ of\ x\ \ that\ will\ make\ A = 0$

$A(-2 + 2) + B(-2 + 3) = 1$

$0 + B = 1$

$\therefore B = 1$

$A(x+2) + B(x+3) = 1$

Let $x = -3$

$A(-3+2) + B(-3+3) = 1$

$-A + 0 = 1$

$$\therefore A = -1$$

Alternatively to find A, equate the coefficient s of x to zero because x does exist in the R.H.S

A+B=0

A=-B

equate constants at L.H.S to one at R.H.S

2A+3B=1

2(-B)+3B=1

-2B+3B=1

$$\therefore B = 1$$

But A=-B

$$\therefore A = -1$$

substituting the values of a and b

$=\frac{A}{x+3} + \frac{B}{x+2}$

$=\frac{-1}{x+3} + \frac{1}{x+2}$

Example 2

Express

$\frac{10(x+1)}{(x+3)(x^2+1)}$ *in partial fraction*

Solution

Note (x^2+1) *cannot be factorized*

$\frac{10(x+1)}{(x+3)(x^2+1)} = \frac{A}{X+3} + \frac{Bx+C}{x^2+1}$

$\frac{10(x+1)}{(x+3)(x^2+1)} = \frac{A(x^2+1)(Bx+C)(x+3)}{(x+3)(x^2+1)}$

Equating the numerators

10(x+1)=$A(x^2+1) + (Bx+C)(x+3)$

Putting x=-3

10(-3+1)=$A(-3^2+1)+(B(-3)+C)(-3+3)$

-20A=10A

$\therefore A=-2$

Equating components or coefficents of x

10(x+1)=$Ax^2+A+Bx^2+Cx+3Bx+3C$

$\therefore A+3C=10\ \ldots..(i)$

A+B=0 …… (ii)

But A=-2

A+3C=10

-2+3C=10

$\therefore 3C=12$

C=4

A+B=0

-2+B=0

$\therefore B=2$

Substituting

$\frac{A}{X+3}+\frac{Bx+C}{x^2+1}$

$\frac{-2}{X+3}+\frac{2x+4}{x^2+1}$

Exercises

Express the function$\frac{7x+2}{(x+2)^2(x-2)}$

MATHEMATICAL INDUCTION

This is a method of proving results provided the complete statement of the result is known

Example 1

Prove by mathematical induction

$1+2+3+\cdots+n=\frac{n(n+1)}{2}$

Solution

$1+2+3+\cdots+n=\frac{n(n+1)}{2}$

Let n=1 substituting in the formula

$\frac{n(n+1)}{2}$

$\frac{(1)(1+1)}{2}$ i.e $\frac{1\times2}{2}$=1 TRUE

Let n=2

$\frac{(2)(2+1)}{2}$ i.e $\frac{2\times3}{2}$=3 True

If it is true for n , it will also be true for n+1.Adding(n+1) on both sides

$$1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

$$1+2+3+\cdots+n+n(n+1)=\frac{n(n+1)}{2}+(n+1)$$

R.H.S

$=(n+1)(\frac{n}{2}+1)$

$=(n+1)\frac{(n+2)}{2}$

$\frac{(n+1)(n+2)}{2}$

$\therefore 1+2+3+\cdots+n=\frac{n(n+1)}{2}$

Example 2

If n is any natural number, then by mathematical induction prove $1^2+3^2+5^2+\cdots+(2n-1)^2=\frac{1}{3}(4n^3-n)$

Solution

Let n=1; it will be $\frac{1}{3}(4-1)$=1 true

N=2 it will be $\frac{1}{3}(4\times8-2)$

$\frac{1}{3}(30)=10\ \ true$

If it is true for n, it will also be true for 2n+1

Adding (2n +1) on both sides

$1^2+3^2+5^2+\cdots+(2n-1)^2+(2n+1)^2=\frac{1}{3}(4n^3-n)+(2n+1)^2$

R.H.S

$=\frac{1}{3}(4n^3-n)+(2n+1)^2$

$\frac{1}{3}(4n^3-n)+4n^2+4n+1$

$=\frac{4n^3-n+12n^2+12n+3}{3}$

$$\frac{(4n^3 + 12n^2 + 12n + 4) - (n + 1)}{3}$$

$$\frac{1}{3}[4(n^3 + 3n^2 + 3n + 1) - (n + 1)]$$

$$\frac{1}{3}[4(n + 1)^3 - (n + 1)]$$

Exercises

1 prove by induction that

$1 + 2 + 3 + \cdots n < \frac{1}{8}(2n + 1)^2$

2) write down the nth term of the series 1.3+2.5+3.7+4.9+… Hence find the sum of the first n terms of the series.

3) find the sum of the series $\sum_{r=n+1}^{2n} r^2$

COMPLEX NUMBER

A Complex number, Z, is any number that can be written in the form z =x+iy, where x and y are real numbers. For example 3+2i is a complex number.x is called the real part of Z written as Re(Z);y is called the imaginary part of Z, written as Im(Z).

NOTE $\sqrt{-1}$=i

therefore $\sqrt{-1} \times \sqrt{-1}$=-1

Reduction of complex number

$$i^3 = i^2 \times i = -1 \times i = -i$$

$$i^5 = i^2 \times i^2 \times i = i$$

Addition or subtraction of complex Numbers

In addition or subtraction of complex numbers, we add or subtract corresponding real parts and imaginary parts.

Question

If $Z_1 = 3 + 4i$

And $Z_2 = 2 - i$

Find (i) $Z_1 + Z_2$=$3 + 4i + 2 - i$=5+3i

(ii) $Z_1 - Z_2$= $(3 + 4i)$- $2 - i$=1+5i

Complex conjugate

If $Z_1 = 3 + 2i$

Then the conjugate of Z normally called Z bar ($\overline{Z}$) is the negation of the imaginary part of the number only.

$\bar{Z}$=3-2i

If Z_1=-2-3i

$\overline{Z_1}$=-2+3i

Multiplication of Complex Numbers

In the multiplication of complex numbers, we follow the algebraic expansion method but must replace i^2 with -1.

Question

If $Z_1 = 3 + 4i$ and $Z_2 = 2 - i$

Find (i)$Z_1 Z_2$ (ii)$\overline{Z_1} Z_2$

Solution

$Z_1 Z_2$= $(3 + 4i\ )(2 - i)$

=6-3i+8i-4i^2
=6+5i-4i^2but i^2=-1
=6+5i-4(-1)
=6+5i+4
$Z_1 Z_2$=10+5i

ii) If Z_1= $3 + 4i, Therefore$ $\overline{Z_1}$=3-4i

ButIf Z_2= $2 - i$,

Then $\overline{Z_1} Z_2$= $(3 - 4i)(2 - i)$

=6-3i-8i+4i^2

=6-11i-4

=2-11i

Division of complex Numbers

In the division of complex Numbers. we rationalize with the conjugate of the denominator. By rationalizing, we mean using the conjugate of the denominator to multiply both numerator and denominator.

Question

If Z_1=3+4i and Z_2=2-I

Find $\frac{Z_2}{Z_1}$ =$\frac{2-i}{3+4i}$

By rationalization

$\frac{2-i}{3+4i} \times \frac{3-4i}{3-4i}$=$\frac{6-8i-3i+4i^2}{9-2i-12i-16i^2}$=$\frac{6-11+4i^2}{9-16i^2}$ but $i^2 = -1$

$$\frac{6-11+4(-1)}{9-16(-1)}$$

$\frac{6-11-4}{9+16}$=$\frac{2}{25}$-$\frac{11}{25}i$

ii)find $\frac{Z_2}{\overline{Z_1}}$=$\frac{2-i}{3-4i}$

By rationalization

$$\frac{2-i}{3-4i} \times \frac{3+4i}{3+4i}$$

=$\frac{(2-i)(3+4i)}{(3-4i)(3+4i)}$=$\frac{6+8i-3i+4i^2}{9+2i-12i-16i^2}$=$\frac{6+5i-4(-1)}{9-16(-1)}$=$\frac{6+5i+4}{25}$=$\frac{10+5i}{25}$=$\frac{10}{25}$+$\frac{5}{25}$I

$\frac{Z_2}{\overline{Z_1}}$=$\frac{2}{5}$+$\frac{1}{5}i$

Exercise

Find the real and imaginary part of $3 + 2i/i$

Modulus or Absolute VALUE

The distance OZ=r is the modulus or absolute value of Z is the distance of Z from the origin O. By Pythagoras' Theorem

hy^2=OP^2+Ad^2

r^2=x^2+y^2

r=$\pm\sqrt{x^2+y^2}$

Modulus=$|r|$=$\sqrt{x^2 + y^2}$

The modulus is the square root of summation of squares of real and imaginary values. For instance

If $Z_1 = 3 + 2i\ find|Z_1|$

SOLUTION

If Z_1=3+2i

Here x=3 and y=2

$\therefore$ $|Z_1|$=3+2i

Hence x=3 and y=2

$|Z_1|=\sqrt{x^2+y^2}$

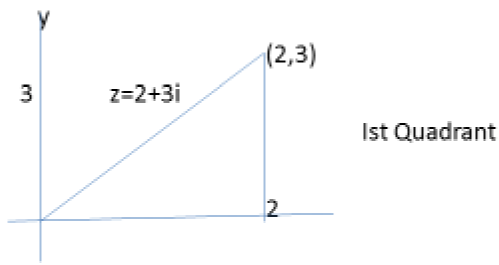
$\sqrt{3^2+2^2}$

$\sqrt{9+4}$

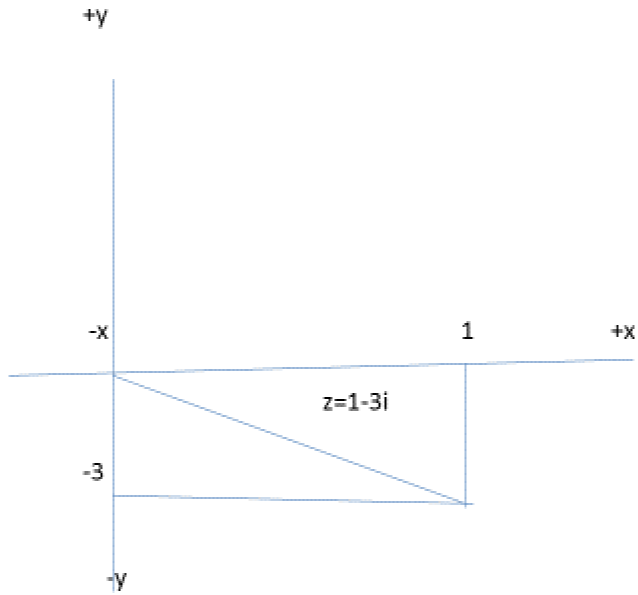
$|Z|=\sqrt{13}$

- Argand Diagram
- Representation of a complex number (x+iy) as a point in plane graphically is termed Argand Diagram
- i z=2+3i

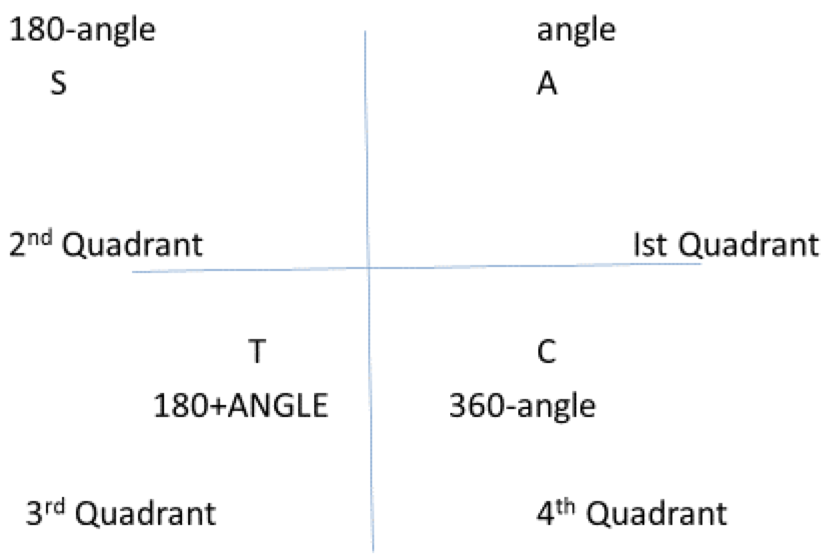
Hence x=2,y=3



- ii) Z=1-3i
- Here x=1,y=-3



- Argument
- The angle, line OZ makes with the positive x-axis is the Argument of the complex number.



I) if the directed line segment is in the first quadrant argZ=$tan - 1\frac{y}{x}$

2) if the directed line segment is in the second quadrant argZ=180-$tan - 1\frac{y}{x}$

3) If the directed line segment in the third quadrant argZ=180+$tan - 1\frac{y}{x}$

4)If the directed line segment in the fourth quadrant argZ=360+$tan - 1\frac{y}{x}$

Question

If $Z_1 = 3 + 4i\ and Z_2 = 2 - i$

PERMUTATIONS (WAYS OR ARRANGEMENTS)

Under permutation order is important since abc is different from acb etc.By permutation we mean number of ways by which elements can be expressed and the symbol ‘p’ is used.

It is expressed thus

$^{n}p_{r}$

$=\frac{n!}{(n-r)!}$

Question

In how many ways can three boys be arranged from five

Solution

Here selection or formation or chosen is not used, thus it is permutation

Total number of boys=5

To arrange =3

$^{n}p_{r}$

$={}^{5}p_{3}$

$=\frac{5!}{(5-3)!}=\frac{5!}{2!}=\frac{5\times4\times3\times2\times1}{2\times1}$=60 ways

solve

1) $6p_2$

(ii) $^{7}p_{3}$

(iii) $^{8}p_{4}$

Combination

Under combination order is irrelevant abc, acb, cab are the same. Hence the word chosen or formed or selected is used in addition to ways. The symbol C is used and it is expressed thus

$^{n}C_{r}$

$=\frac{Upper}{lower!(difference)!}=\frac{n!}{r!(n-r)!}$

question

In how many ways can four boys be chosen from six

Solution

Here chosen is used and thus it is combination

Total number of boys=6

To choose=4

$^{n}C_{r} = {}^{6}C_{4} = \frac{6!}{4!(6-4)!} = \frac{6\times5\times4!}{4!\times2\times1}$=15 ways

A committee of two men and three women is to be chosen from six men and four women

a) How many different committee can be formed.

b) If one of the women refuses to serve on the same committee as a particular man,how many committees are now possible.

Solution

men	women
6	4
2	3

${}^{6}C_{2} \quad \times \quad {}^{4}C_{3}$

$\frac{6!}{2!4!} \times \frac{4!}{3!1!}$=15$\times$ $4 = 60\ committees$

Questions

ONE WOMAN Refused to serve. Also one man refused to serve. This will affect the total number and also the committee

Total 6 4 2 3

- 1 1 1 1
- 5 3 1 2
- The remaining of the committee to be selected from the remaining of the total people
- **MEN** **WOMEN**
- ${}^{5}C_{1}$ ${}^{3}C_{2}$
- ${}^{5}C_{1}$ X ${}^{3}C_{2}$

=15 Ways

60-15=45 ways

CLASSWORK

In how many distinct ways can 10 black,5 red and 8 yellow balls be arranged in a straight line

In how many ways can 3 balls be selected from question (a) above so that

I all the colours are there

II one colour is there

Binomial Theorem

Any expression of the form $(a+x)^n$ involving two terms is known as a binomial function and the statement of its expansion in power of x is known as the binomial theorem.

$(1+x)^1$=1+x

$(1+x)^2 = 1+2x+x^2$

$(1+x)^3$=1+3x+$3x^2$+x^3

$(1+x)^4$=1+4x+$6x^2$+$4x^3$+x^4

$(1+x)^5$=1+5x+$10x^2$+$10x^3$+$5x^4$+x^5

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

Question

Write down in its simplest form the complete expansion of $(x - \frac{1}{2})^6$ *by taking* $x = \frac{1}{400}$ in this expansion

2) Expand (x-2y)^{3}using pascal triangle

3) Expand (1+3x)^3 *using binomial theorem*.

Solution

1) Expand (1+3x)^3 *using binomial theorem*.

1+ 3(1)(3x)^2+3(1)(3x^3)+(3x^4)

1+27$x^2 + 243x^3 + 81x^4$

2) Expand (x-2y)^{3}using pascal triangle

$$x^3 - 3(x^2)(2y) + 3x(2y)^2 - (2y)^3$$
$$x^3 - 6x^2y + 12xy^2 - 8y^3$$

Exercise

Write down in its simplest form the complete expansion of $(x - \frac{1}{2})^6$ $by\ taking\ x = \frac{1}{400}$ in this expansion

TRIGONOMETRY

The ratio of the length of a two sides of a right angle angle

There are basically two definition namely

i)The traditional and modern definition

$\frac{opp}{hyp} = sin\theta$

$\frac{Adj}{Hyp}$=cos θ

$\frac{Opp}{Adj} = Tan\theta$

$Hyp^2 = Opp^2 + Adj^2$ Pythagoras

Reciprocal of angles

$\frac{1}{sin\theta}$=cosecantθ

$\frac{1}{Cos\theta}$=Secantθ

Relationship of the Ratios

$\frac{Sin\theta}{Cos\theta}$=Tan$\theta$.....(i)

$Sin^2\theta + Cos^2\theta$=1.....(ii)

Question

Show that $\frac{Sin\theta}{Cos\theta}$=Tan$\theta$

L.H.S

$\frac{OP}{HP} \div \frac{Ad}{Hy}$

$\frac{OP}{HP} \div \frac{Hy}{Ad}$

$\frac{OPP}{Ad}$=$Tan\theta$

$\therefore \frac{Sin\theta}{Cos\theta}$=$Tan\theta$

Required to show that

$Sin^2\theta + Cos^2\theta = 1$

L.H.S

$(\frac{OP}{HY})^2 + (\frac{Ad}{Hy})^2$

Finding the L.C.M

$\frac{Op^2+Ad^2}{Hy^2}$=

But $Hy^2 = Op^2 + Ad^2$ by Pythagoras Theorem

Substituting Hy^2 in place of the numerator $Op^2 + Ad^2$ gives $\frac{Hy^2}{Hy^2} = 1$

$Sin^2\theta + Cos^2\theta = 1$

Diving through by $Cos^2\theta$

$\frac{Sin^2\theta}{Cos^2\theta}$+$\frac{Cos^2\theta}{Cos^2\theta}$=$\frac{1}{Cos^2\theta}$

$Tan^2\theta + 1 = Sec^2\theta$

$1 + Tan^2\theta$=$Sec^2\theta$

Also if$Sin^2\ \theta + Cos^2\theta = 1$

Dividing through by $Sin^2\theta$

$\frac{Sin^2\theta}{Sin^2\theta}$+$\frac{Cos^2\theta}{Sin^2\theta}$=$\frac{1}{Sin^2\theta}$

$1 + cot^2\theta$=cos$ec^2\theta$

Question

If $Sin^2\theta = \frac{4}{5}\ find\ (1)Cos\theta\ (ii)Tan\theta\ (iii)Sec\theta$

$Sin^2\ \theta + Cos^2\theta = 1$

$(\frac{4}{5})^2$+$Cos^2\theta = 1$

$\frac{16}{25}$+$Cos^2\theta = 1$

$Cos^2\theta = 1 - \ \frac{16}{25}$

$Cos^2\theta$=$\frac{9}{25}$

cos$\theta = \pm\sqrt{\frac{9}{25}}$

cosθ=$\frac{3}{5}$

Eliminate $\theta\ between\ the\ equation\ x = tan\theta +$ cot θ,y=sin$\theta - cos\theta$

Solution

x=$\frac{tan\theta}{1}$+$\frac{1}{tan\theta}$

x=$\frac{tan^2\theta+1}{tan\theta}$=$\frac{sec^2\theta}{tan\theta}$

x=$sec^2\theta \div tan\theta$

x=$\frac{1}{cos^2\theta} \div \frac{sin\theta}{cos\theta}$

- $\frac{x}{1}$=$\frac{1}{cos\theta sin\theta}$
- Taking reciprocal of the equation $\frac{1}{x} = sin\theta cos\theta$
- Addition formula or Double Angle
- Sin(A+B)=Sin A Cos B +Cos A Sin B
- Sin(A-B)=Sin A Cos B- Cos A Sin B
- Cos (A+B)=Cos A Cos B-Sin A Sin B
- Cos (A-B)=Cos A Cos B+ Sin A Sin B

Prove that $\frac{1+cosx}{1-cosx}$+$\frac{1-cosx}{1+cosx}$=4$cot^2x + 2$

$$\frac{(1+cosx)(1+cosx)+(1-cosx)(1-cosx)}{(1-cosx)(1+cosx)}$$

$$\frac{1+cosx+cosx+cos^2x+1+cosx-cosx-cos^2x}{1+cosx-cosx-cos^2x}$$

$$\frac{2+2cos^2x}{1-cos^2x}$$

$$\frac{2+2cos^2x}{sin\,^2x}$$

$$\frac{2}{sin\,^2x}+\frac{2cos^2x}{sin\,^2x}$$

$$2[\frac{1}{sin\,^2x}]+2cot^2x]$$

$$2cot^2x+2cot^2x+1$$

$4cot^2x + 1$

- **Question**
- If Sin $75^0 = 0.9$, $Sin15^0 = 0.1$, $Cos\ 75^0 = 0.2$, $Cos15^0$=0.9
- Find (i) sin60^0 $and\ (ii) \cos 60^0$
- Solution
- i) sin60^0=Sin($75^0 - 15^0$)

  ASSIGNNMENT

  Use pascal triangle to solve (x-2y) 2
- Prove that $\frac{1+cosx}{1-cosx}$+$\frac{1-cosx}{1+cosx}$=4$cot^2x + 2$
- Eliminate $\theta\ between\ the\ equation\ x = tan\theta + \cot\ \theta$,y=sin$\theta - cos\theta$
- In how many ways can four boys be chosen from six
-